Decoherence Rate of Semiconductor Charge Qubit

Presenter: Leonid Fedichkin

In collaboration with Arkady Fedorov

Principal Investigator: Vladimir Privman

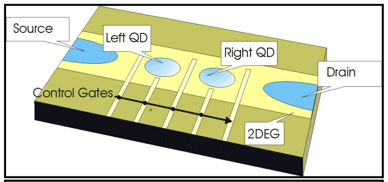
Center for Quantum Device Technology
Clarkson University, Potsdam, NY

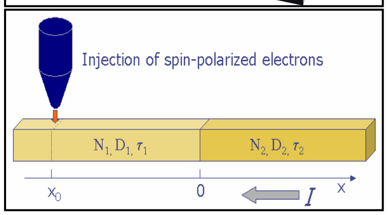
NSF-DMR-0121146, ITR/SY: Center for Modeling of Quantum Dynamics, Relaxation and Decoherence in Solid-State Physics for Information-Technology Applications

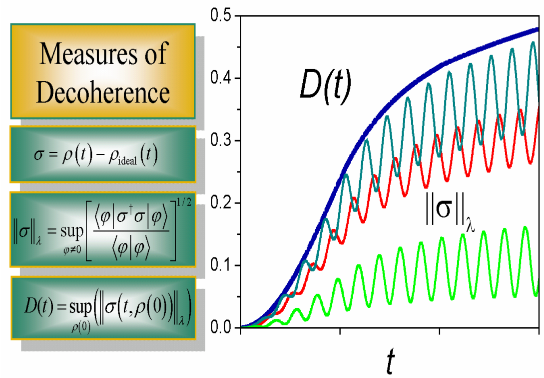
PI: Vladimir Privman, Institution: Clarkson University

Research Objectives

The main objectives of our program have been to explore coherent quantum mechanical processes in novel solid-state semiconductor information processing devices with components of atomic dimensions. These include quantum computers, spintronic devices, and nanometer-scale logic gates.







Significant Results

Our achievements to date include:

- ✓ new measures of initial decoherence, and evaluation of decoherence for spins in semiconductors;
- ✓ evaluation of solid-state quantum computing designs;
- ✓ studies of transport associated with quantum measurement;
- ✓ investigation of spin-polarized devices and role of nuclear spins in spintronics and quantum computing;
- ✓ general contributions to quantum computing algorithms and to time-dependent and phase-related properties of open many-body quantum mechanical systems;
- ✓ novel analytical and numerical Monte Carlo approaches to studying spin-polarization control for spintronic device modeling;
- ✓ investigation of spin relaxation dynamics in two-dimensional semiconductor heterostructures.

Approach

Our approach has been truly interdisciplinary. For example, in developing new measures of decoherence for quantum computing, we have employed concepts from many-body quantum physics, computer error-correction algorithms, and nonequilibrium statistical mechanics. In our description of spintronic devices, we have utilized large-scale Monte Carlo simulations, knowledge from solid-state physics of semiconductors and from the microelectronics area of electrical engineering, as well as novel ideas of coherent control of quantum dynamics.

Our approach has been to design and evaluate architectures that allow implementation of many gate cycles during the relaxation and decoherence times. This requires development of techniques to evaluate all the relevant time scales: single- and two-quantum-bit gate “clock” times, as well as time scales of relaxation processes owing to the quantum bit (e.g., spin) interactions with environment, such as phonons or surrounding spins. We have also studied spin-control and charge carrier transport for spintronics and quantum measurement.

Broader Impact

We have extensive research *collaborations* with leading experimental and theoretical groups. The *educational impact* has included training undergraduate and graduate students, postdoctoral researchers, and development of three new courses to introduce quantum device and quantum algorithmic concepts to graduate and undergraduate students. Our program has contributed to *homeland security* and received funding from the National Security Agency.

Our outreach program has included sponsoring presentation events, and an international workshop series *Quantum Device Technology*, held in May of 2002 and May 2004, and sponsored by the Nanotechnology Council of IEEE and NSA (via ARO). We have worked with the REU site for students at SUNY Potsdam to guide several *undergraduate research projects* in the topics of quantum computing and quantum algorithms.

References

Experimental realizations:

- T. Hayashi, T. Fujisawa, H.-D. Cheong, Y.-H. Jeong and Y. Hirayama, *Phys. Rev. Lett.* **91**, 226804 (2003).
- L.C.L. Hollenberg, A.S. Dzurak, C. Wellard, A.R. Hamilton, D.J. Reilly, G.J. Milburn and R.G. Clark, *Phys. Rev. B* **69**, 113301 (2004).

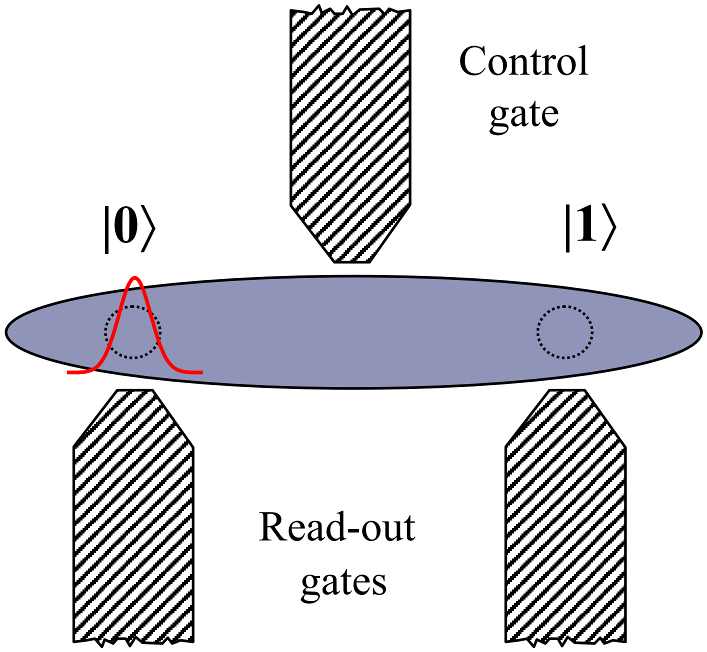
Theory:

- L. Fedichkin, M. Yanchenko and K.A. Valiev, *Nanotechnology* **11**, 387 (2000).
- S. A. Gurvitz, L. Fedichkin, D. Mozyrsky, G. P. Berman, *Phys. Rev. Lett.* **91**, 066801 (2003).
- L. Fedichkin and A. Fedorov, *Phys. Rev. A* **69**, 032311 (2004).
- A. Fedorov, L. Fedichkin, and V. Privman, e-prints cond-mat/0401248, cond-mat/0309685, cond-mat/0303158.

Semiconductor Quantum Dot Charge Qubit

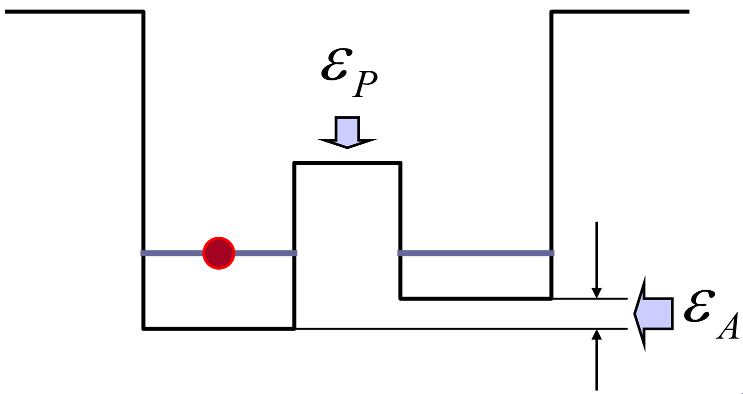
Two-level approximation for an electron in double-well

$$\boxed{H_e = -\frac{1}{2}\varepsilon_A(t)\sigma_X - \frac{1}{2}\varepsilon_P(t)\sigma_Z, \text{ in the basis } \{|0\rangle, |1\rangle\}}$$



Energy splitting: $\varepsilon = \sqrt{\varepsilon_A^2 + \varepsilon_P^2}$

$$\psi(r) = \alpha\psi_L(r) + \beta\psi_R(r)$$

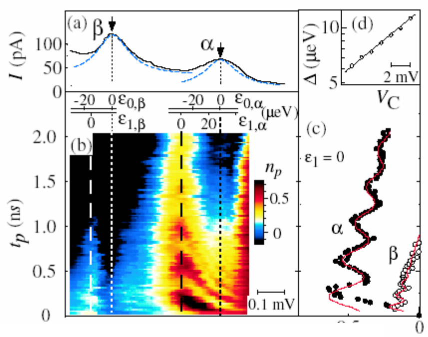


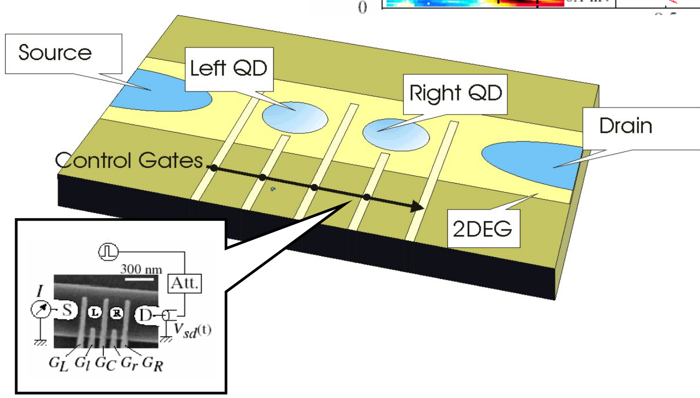
L. Fedichkin et al., *Nanotechnology* **11**, 387 (2000).

Experimental Realizations

Gate-engineered Quantum Double-Dot in GaAs

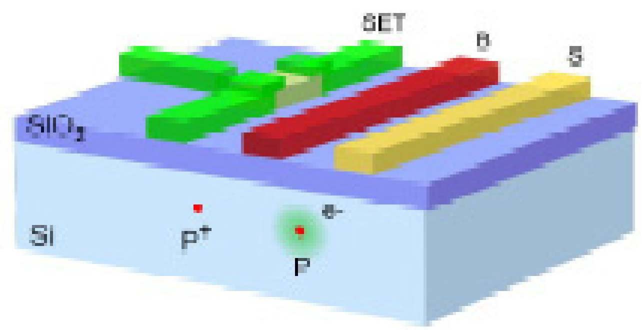
T. Hayashi, T. Fujisawa, H.-D. Cheong, Y.-H. Jeong, Y. Hirayama, Phys. Rev. Lett. **91**, 226804 (2003)





Electron in Coupled Double-Phosphorus Impurity in Si

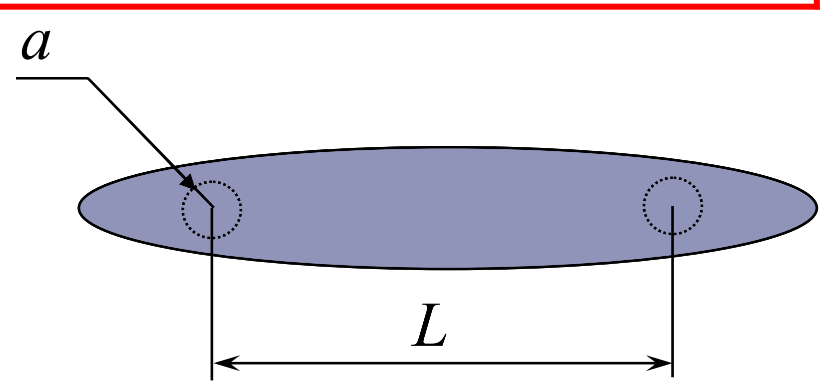
L.C.L. Hollenberg, A.S. Dzurak, C. Wellard, A.R. Hamilton, D.J. Reilly, G.J. Milburn, and R.G. Clark, Phys. Rev. B **69**, 113301 (2004) ;



Main Parameters

Parameter	Gate-engineered DD in GaAs, Si	Double P-impurity in Si
L	50 nm	
a	25 nm	3 nm
Ψ(r)	Gaussian ~exp($-r^2/(2a^2)$)	Hydrogenic ~exp(-r/a)
T	0	

L – distance between the electron density maxima;
a – dot radius.



Maximal Deviation norm D(t)

We define a deviation from the ideal (without phonons) density operator according to

$$\sigma(t) \equiv \rho(t) - \rho_{\text{ideal}}(t); \qquad \rho_{\text{ideal}}(t) = e^{-iH_e t}\rho(0)\,e^{iH_e t}.$$

As a *numerical measure* we use an operator norm

$$\|\sigma\|_\lambda = \max_i |\lambda_i|,$$

In case of two-level system it is $\|\sigma\|_\lambda = \sqrt{|\sigma_{00}|^2 + |\sigma_{01}|^2}.$

It not only depends on time but also on the initial density matrix ρ(0).
The effect of phonons can be better quantified by *D(t)*

$$\boxed{D(t) = \sup_{\rho(0)}\left(\|\sigma(t,\rho(0))\|_\lambda\right)}$$

Should be compared to fault-tolerant thresold

$$D(\Delta t) \le O(10^{-4})$$

Hamiltonian of the Semiconductor Qubit Coupled to Acoustic Phonons

$$H = H_e + H_p + H_{ep}$$

$$H_e = -\frac{1}{2}\varepsilon_A(t)\sigma_X - \frac{1}{2}\varepsilon_P(t)\sigma_Z$$

$$H_p = \sum_{\mathbf{q},\lambda} \hbar s q b_{\mathbf{q},\lambda}^{\dagger} b_{\mathbf{q},\lambda}$$

$$H_{ep} = \sum_{\mathbf{q},\lambda} \left(g_{\mathbf{q},\lambda} b_{\mathbf{q},\lambda}^{\dagger} + g_{\mathbf{q},\lambda}^{*} b_{\mathbf{q},\lambda} \right) \sigma_Z$$

Electron-Phonon Coupling

The Gate-engineered Double Dot

$$g_{\mathbf{q}} = iq\Xi\sqrt{\frac{\hbar}{2\rho s q V}}e^{-\frac{a^2q^2}{4}-i\mathbf{qR}}\sin\left(\frac{\mathbf{qL}}{2}\right),\ \text{for DA phonons}$$

$$g_{\mathbf{q}} = -\frac{M}{q^2}\sqrt{\frac{\hbar}{2\rho s q V}}\left(\xi_1 q_2 q_3 + \xi_2 q_3 q_1 + \xi_3 q_1 q_2\right)e^{-\frac{a^2q^2}{4}-i\mathbf{qR}}\sin\left(\frac{\mathbf{qL}}{2}\right),\ \text{for PA phonons}$$

The Double-Impurity

$$g_{\mathbf{q}} = iq\Xi\sqrt{\frac{\hbar}{2\rho s q V}}\frac{e^{-i\mathbf{qR}}}{(1+a^2q^2/4)^2}\sin\left(\frac{\mathbf{qL}}{2}\right),\ \text{for DA phonons}$$

Emission of the Phonon (the NOT-gate)

$$\boxed{H_e = -\frac{1}{2}\varepsilon_P \sigma_Z,\ \varepsilon = \varepsilon_P = \text{const};\ \ \varepsilon_A\left(t\right) = 0,\quad \text{for } t \in \left[0, 2\pi/\varepsilon\right]}$$

In the basis $\{|-\rangle, |+\rangle\}$ the density matrix will have the form

$$\rho(\Delta t) = \begin{pmatrix} 1 - \rho_{--}(0)e^{-\Gamma\Delta t} & \rho_{+-}(0)\exp\left[-\left(\frac{\Gamma}{2} - i\frac{\varepsilon}{\hbar}\right)\Delta t\right] \\ \rho_{-+}(0)\exp\left[-\left(\frac{\Gamma}{2} + i\frac{\varepsilon}{\hbar}\right)\Delta t\right] & \rho_{--}(0)e^{-\Gamma\Delta t} \end{pmatrix}$$

$\Gamma -$ is the relaxation rate, obtained within Fermi Golden Rule

Error (the NOT Gate)

$$D_A(\Delta t)\Big|_{DA} = D_A(\Delta t)\Big|_{IDA} = \frac{\Xi^2 L^2 \varepsilon^5}{24\pi\rho s^7 \hbar^6};$$

$$D_A(\Delta t)\Big|_{PA} = \frac{M^2 L^2 \varepsilon^3}{24\pi\rho s^5 \hbar^4}.$$

Here Ξ is the deformational potential, M is the piezo-constant, ρ is the density of the crystal, and s is the speed of the sound.

DA stands for the deformational phonons and the gate-engineered double-dot;

IDA stands for the deformational phonons and the double-impurity;

PA stands for the piezo-phonons and the gate-engineered double-dot.

Dephasing (the π-PHASE Gate)

$$\boxed{H_e = -\frac{1}{2}\varepsilon_A \sigma_X,\ \varepsilon = \varepsilon_A = \text{const}; \varepsilon_P(t) = 0, \quad \text{for } t \in [0, 2\pi/\varepsilon]}$$

In the basis $\{|0\rangle, |1\rangle\}$ the density matrix will have the form

$$\rho(\Delta t) = \begin{pmatrix} \rho_{00}(0) & \rho_{01}(0)\exp\left(-B^2(\Delta t) + i\frac{\varepsilon}{\hbar}\Delta t\right) \\ \rho_{10}(0)\exp\left(-B^2(\Delta t) - i\frac{\varepsilon}{\hbar}\Delta t\right) & \rho_{11}(0) \end{pmatrix}$$

The evolution is determined by the spectral function $B^2(t)$, which is in our case:

$$B^2(t) = \frac{8}{\hbar^2} \sum_{\mathbf{q},\lambda} \frac{|g_{\mathbf{q},\lambda}|^2}{s^2 q^2} \sin^2 \frac{sqt}{2}$$

Error (the π-PHASE Gate)

$$D_P\left(\Delta t\right)\Big|_{DA} = \frac{\Xi^2}{2\pi^2\hbar\rho s^3 a^2} \qquad D_P\left(\Delta t\right)\Big|_{PA} = \frac{M^2 L^2}{60\pi^2\hbar\rho s^3 a^2}$$

$$D_P\left(\Delta t\right)\Big|_{IDA} = \frac{\Xi^2}{3\pi^2\hbar\rho s^3 a^2}$$

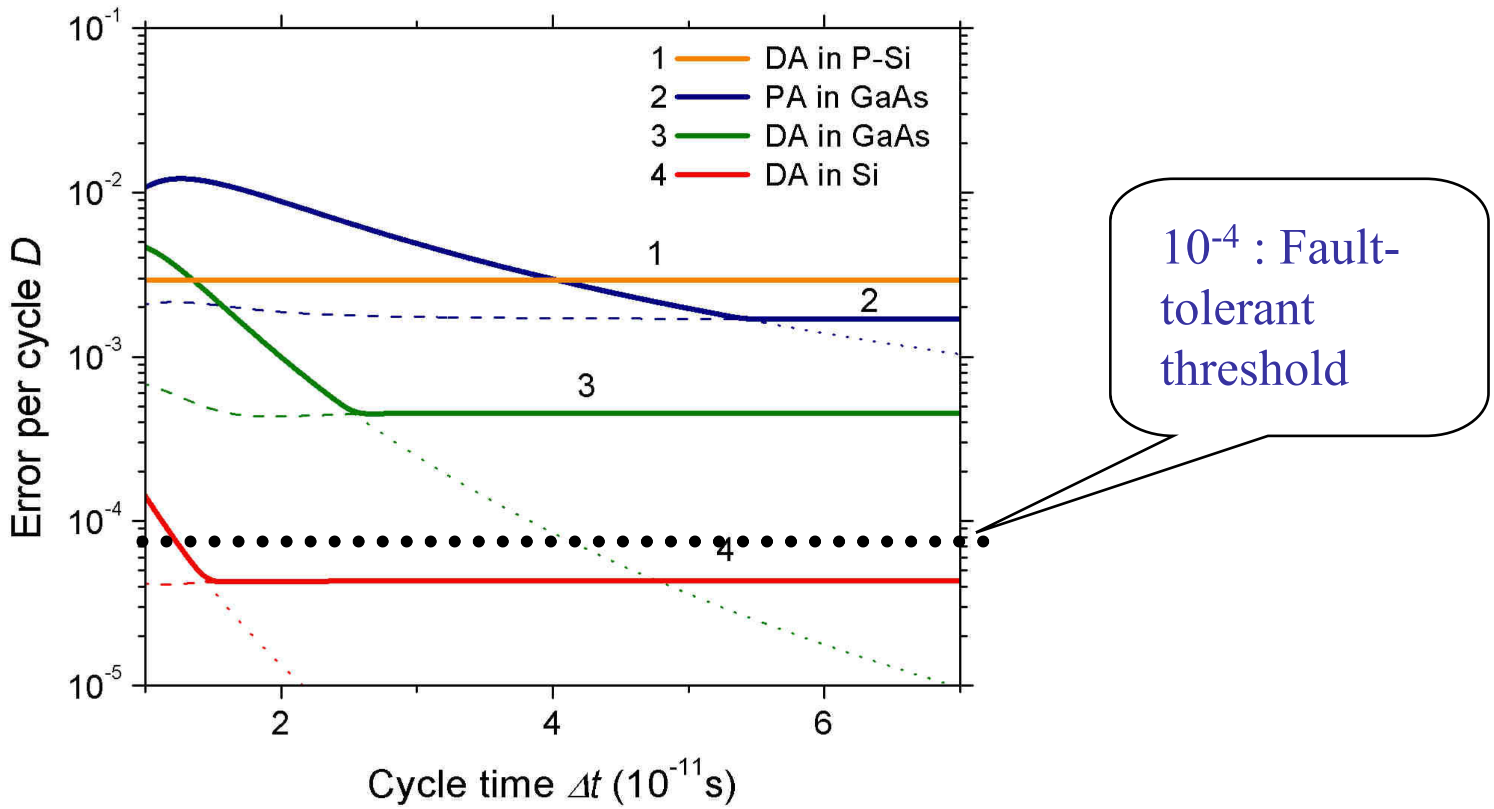
DA stands for the deformational phonons and the gate-engineered double-dot;

IDA stands for the deformational phonons and the double-impurity;

PA stands for the piezo-phonons and the gate-engineered double-dot.

Estimate of the Error Rate per Cycle due to the Electron-Phonon Interaction as a Function of the Cycle Time t ($t = \pi\hbar/\varepsilon$)

Notation : DA – deformational acoustic phonons; PA – piezo-acoustic phonons



L. Fedichkin and A. Fedorov, Phys. Rev. A **69**, 032311 (2004)

Summary

- Acoustic phonon assisted decoherence rates of the semiconductor charge qubit were obtained
- Dephasing appears to be the limiting factor of the qubit performance
- Decoherence rate for the gate-engineered GaAs and Si coupled quantum dots with flexible geometric parameters can be controlled more coherently