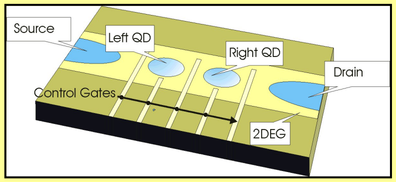
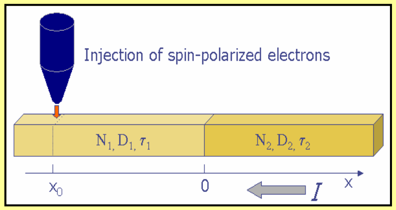
NSF-DMR-0121146, ITR/SY: Center for Modeling of Quantum Dynamics, Relaxation and Decoherence in Solid-State Physics for Information-Technology Applications

PI: Vladimir Privman, **Institution:** Clarkson University

Research Objectives

The main objectives of our program have been to explore coherent quantum mechanical processes in novel solid-state semiconductor information processing devices with components of atomic dimensions. These include quantum computers, spintronic devices, and nanometer-scale logic gates.



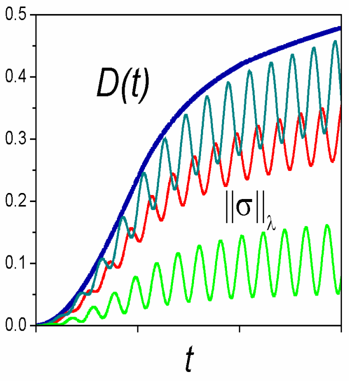


Measures of Decoherence

$$\sigma = \rho(t) - \rho_{\text{ideal}}(t)$$

$$\|\sigma\|_\lambda = \sup_{\varphi \neq 0} \left[\frac{\langle \varphi | \sigma^\dagger \sigma | \varphi \rangle}{\langle \varphi | \varphi \rangle} \right]^{1/2}$$

$$D(t) = \sup_{\rho(0)} \left(\left\| \sigma\left(t, \rho(0)\right) \right\|_\lambda \right)$$



Approach

Our approach has been truly interdisciplinary. For example, in developing new measures of decoherence for quantum computing, we have employed concepts from many-body quantum physics, computer error-correction algorithms, and nonequilibrium statistical mechanics. In our description of spintronic devices, we have utilized large-scale Monte Carlo simulations, knowledge from solid-state physics of semiconductors and from the microelectronics area of electrical engineering, as well as novel ideas of coherent control of quantum dynamics.

Our approach has been to design and evaluate architectures that allow implementation of many gate cycles during the relaxation and decoherence times. This requires development of techniques to evaluate all the relevant time scales: single- and two-quantum-bit gate "clock" times, as well as time scales of relaxation processes owing to the quantum bit (e.g., spin) interactions with environment, such as phonons or surrounding spins. We have also studied spin-control and charge carrier transport for spintronics and quantum measurement.

Significant Results

Our achievements to date include:

- ✓ new measures of initial decoherence, and evaluation of decoherence for spins in semiconductors (**the PI's talk**);
- ✓ evaluation of solid-state quantum computing designs (**poster**);
- ✓ studies of transport associated with quantum measurement;
- ✓ investigation of spin-polarized devices and role of nuclear spins in spintronics and quantum computing (**poster**);
- ✓ general contributions to quantum computing algorithms and to time-dependent and phase-related properties of open many-body quantum mechanical systems;
- ✓ novel analytical and numerical approaches to studying spin-polarization control for spintronic device modeling (**poster**);
- ✓ investigation of spin relaxation dynamics in two-dimensional semiconductor heterostructures (**poster**).

Broader Impact

We have extensive research *collaborations* with leading experimental and theoretical groups. The *educational impact* has included training undergraduate and graduate students, postdoctoral researchers, and development of three new courses to introduce quantum device and quantum algorithmic concepts to graduate and undergraduate students. Our program has contributed to *homeland security* and received funding from the National Security Agency.

Our outreach program has included sponsoring presentation events, and an international workshop series *Quantum Device Technology*, held in May of 2002 and May 2004, and sponsored by the Nanotechnology Council of IEEE and by NSA (via ARO). We have worked with the REU site for students at SUNY Potsdam to guide several *undergraduate research projects* in the topics of quantum computing and quantum algorithms.

Short-Time Decoherence

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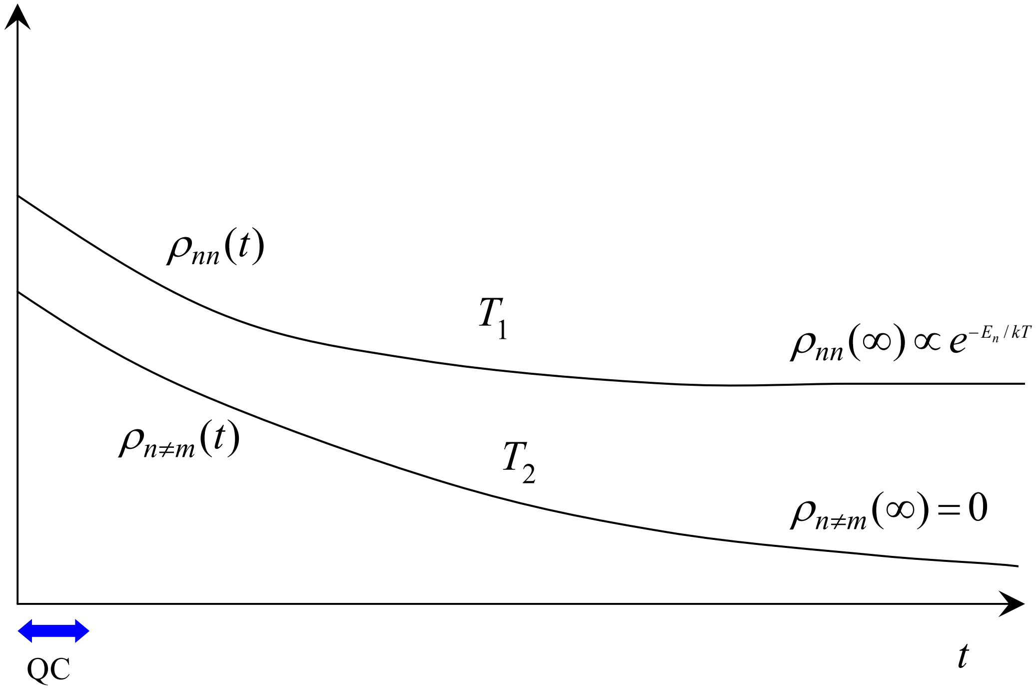
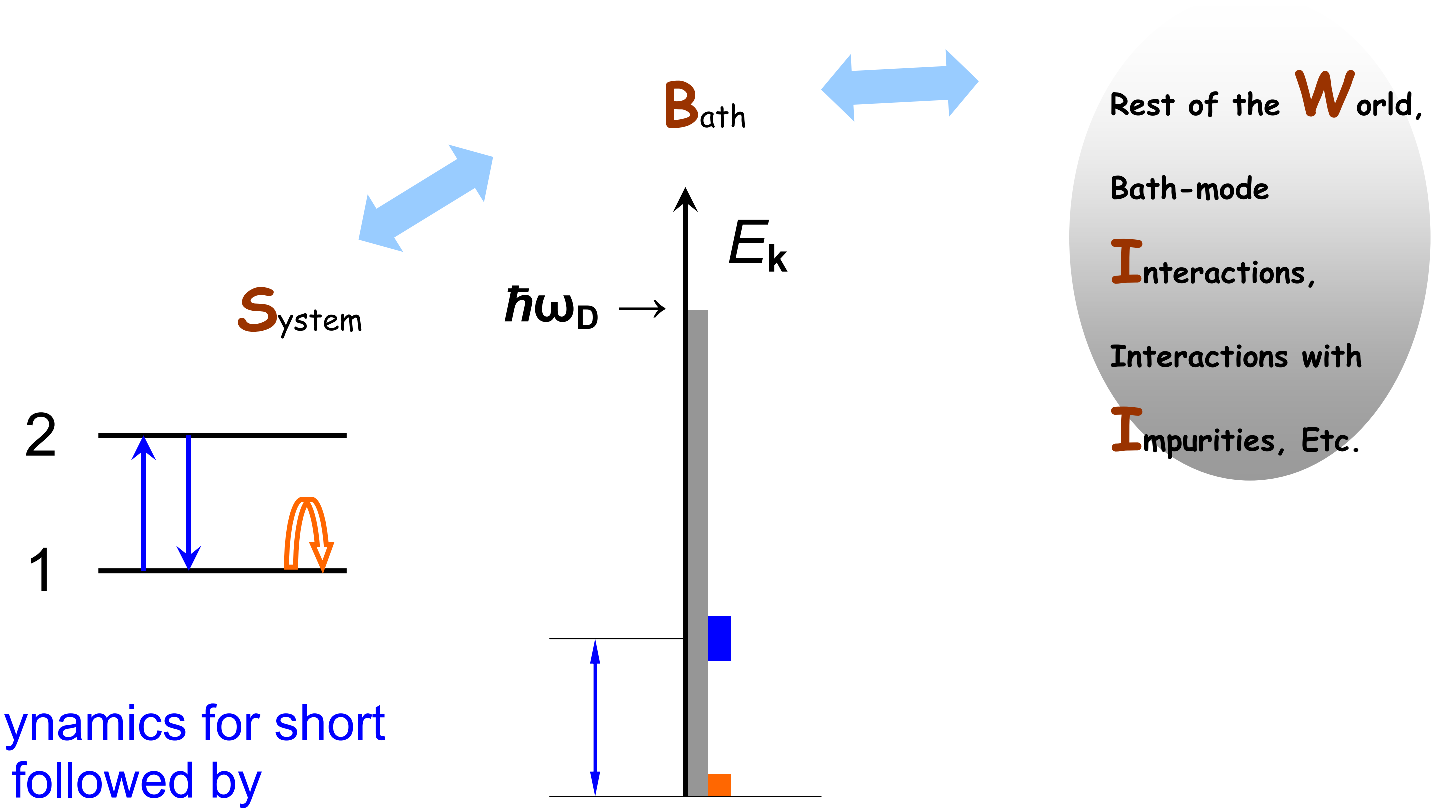
Deviation from Pure/Ideal Quantum States: Can we escape the T_1-T_2 paradigm?

V. Privman: J. Stat. Phys. **110**, 957-970 (2003); **D. Tolkunov** & V. Privman: cond-mat/0403348



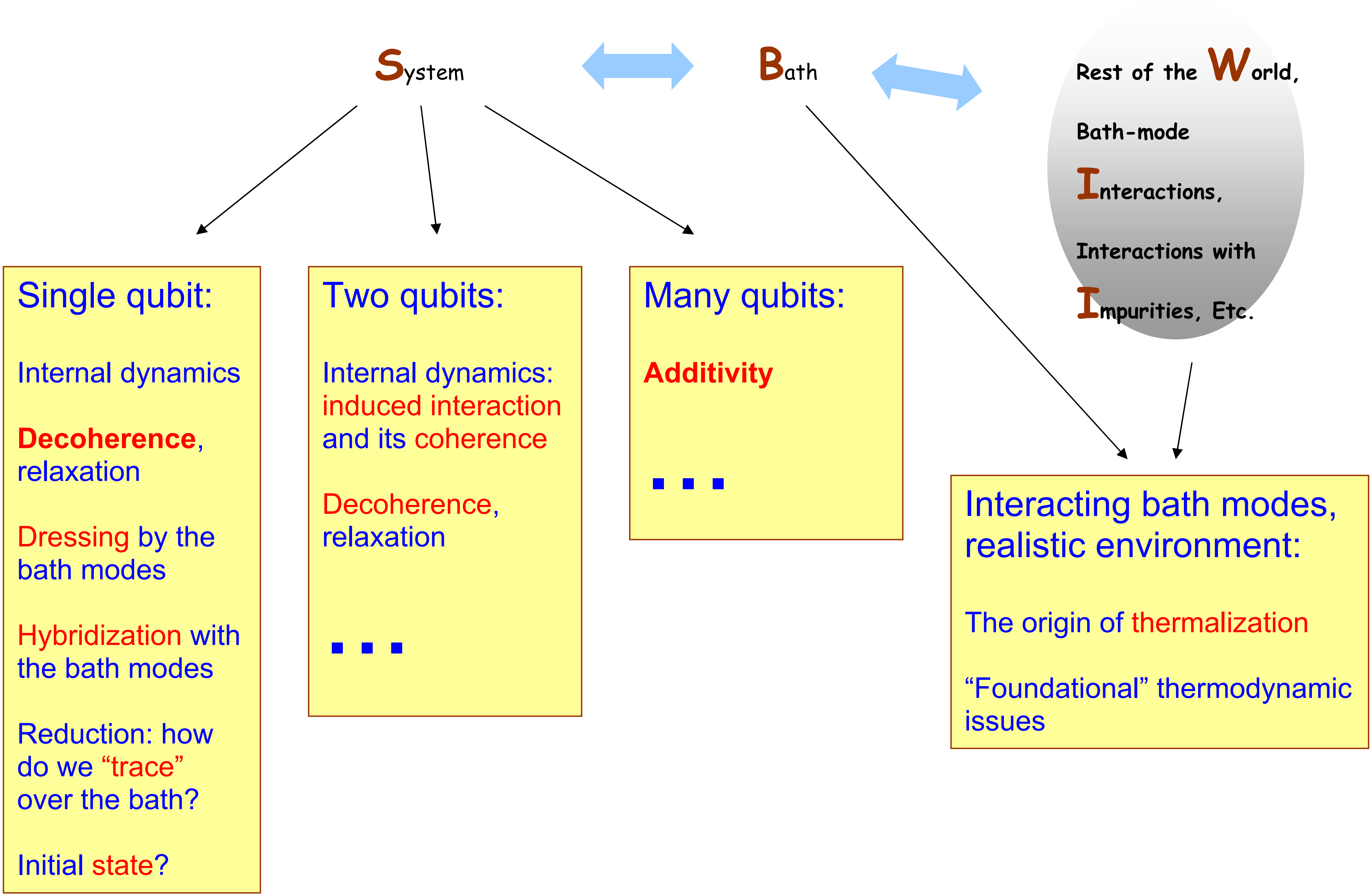
We have developed new short-time approximation schemes for evaluation of decoherence. At low temperatures, the approximation is argued to apply at intermediate times as well. It then provides a tractable approach complementary to Markovian approximations, and is appropriate for evaluation of deviations from pure states in quantum computing models.



Quantum dynamics for short time steps, followed by Markovian approximation, etc.:

$$\frac{1}{T_2} = \frac{1}{2T_1} + \text{pure decoherence}$$

Valid only for $t >> \hbar / kT$



System

Bath

Rest of the World,

Bath-mode

Interactions,

Interactions with

Impurities, Etc.

Single qubit:

Internal dynamics

Decoherence, relaxation

Dressing by the bath modes

Hybridization with the bath modes

Reduction: how do we “trace” over the bath?

Initial state?

Two qubits:

Internal dynamics: induced interaction and its coherence

Decoherence, relaxation

. . .

Many qubits:

Additivity

. . .

Interacting bath modes, realistic environment:

The origin of thermalization

“Foundational” thermodynamic issues

Short-Time Approximation for the Density Matrix

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$$\sqrt{\pi}\,\rho(t) = \int_{-\infty}^{\infty} dy\, e^{-y^2} e^{-iH_S t/2} e^{i[yB(t)\Lambda_S - C(t)\Lambda_S^2]} e^{-iH_S t/2} \rho(0)\, e^{iH_S t/2} e^{-i[yB(t)\Lambda_S - C(t)\Lambda_S^2]} e^{iH_S t/2}$$

$$B^2(t) = 8\sum_K \frac{|g_K|^2}{\omega_K^2}\sin^2\frac{\omega_K t}{2}\coth\frac{\beta\omega_K}{2}$$

$$C(t) = \sum_K \frac{|g_K|^2}{\omega_K^2}\left(\omega_K t - \sin\omega_K t\right)$$

For $\rho(0) = |\uparrow\rangle\langle\uparrow|$ or $|\downarrow\rangle\langle\downarrow|$, we get

$$\mathrm{Tr}_S\,[\rho^2(t)] = \frac{1}{2}[1 + e^{-2B^2(t)}]$$

For $\rho(0) = |\psi_0\rangle\langle\psi_0|$, the deviation from a pure state is apparent: $\rho(t > 0)$ is obviously a *mixture* (integral over y) of pure-state projectors $|\psi(y,t)\rangle\langle\psi(y,t)|$, where

$$|\psi(y,t)\rangle = e^{-iH_S t/2} e^{i[yB(t)\Lambda_S - C(t)\Lambda_S^2]} e^{-iH_S t/2} |\psi_0\rangle$$

Measures of Decoherence

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- **Quantum entropy** $S(t) = -\mathrm{Tr}\left(\rho \ln \rho\right)$, and **idempotency defect**, called also the first order entropy, $s(t) = 1 - \mathrm{Tr}\left(\rho^2\right)$. Both expressions are basis independent, have a minimum, 0, at pure states and measure the degree of the state's "purity."

- **Fidelity** $F(t) = \mathrm{Tr}\left[\rho_{\text{ideal}}(t)\rho(t)\right]$, where $\rho_{\text{ideal}}(t)$ represents the pure-state evolution of the system without the environment. The fidelity attains its maximal value, 1, provided $\rho(t) = \rho_{\text{ideal}}(t)$.

Measures of Decoherence

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- **Norm of deviation:** a new measure of decoherence, introduced in our recent works: **L. Fedichkin**, **A. Fedorov** and V. Privman, Proc. SPIE **5105**, 243-254 (2003) and cond-mat/0309685

- Measure deviation from the ideal evolution by a norm of the deviation operator $\sigma(t) \equiv \rho(t) - \rho_{\text{ideal}}(t)$. We can use the **eigenvalue norm**, $\|\sigma\|_\lambda = \max_i |\lambda_i|$, etc. It has its minimal value, 0, for $\rho(t) = \rho_{\text{ideal}}(t)$, even if the latter is not a projection operator.

The Maximal Norm and Its Properties

To characterize decoherence for an arbitrary initial state, pure or mixed, we define the maximal norm, D, which is determined as norm-of-deviation maximized over all initial density matrices:

$$D(t)=\sup_{\rho(0)}\left(\left\|\sigma(t,\rho(0))\right\|_{\lambda}\right).$$

D is **approximately additive** for weakly interacting qubits, as long as it is small (close to 0) for each, namely **for short times**. This is similar to the approximate additivity of relaxation rates for weakly interacting qubits at large times.

The Maximal Norm and Its Properties

Technical point: why are norms like these candidate for "additivity"? The answer: their definition resembles that of length.

We used $\|\sigma\|_\lambda = \max_i |\lambda_i|$, with σ given by the difference of two density matrices.

A more rigorous definition: $\|A\| = \sup_{\varphi \neq 0} \left[\frac{\langle \varphi | A^\dagger A | \varphi \rangle}{\langle \varphi | \varphi \rangle} \right]^{1/2}$.

Measures of Decoherence

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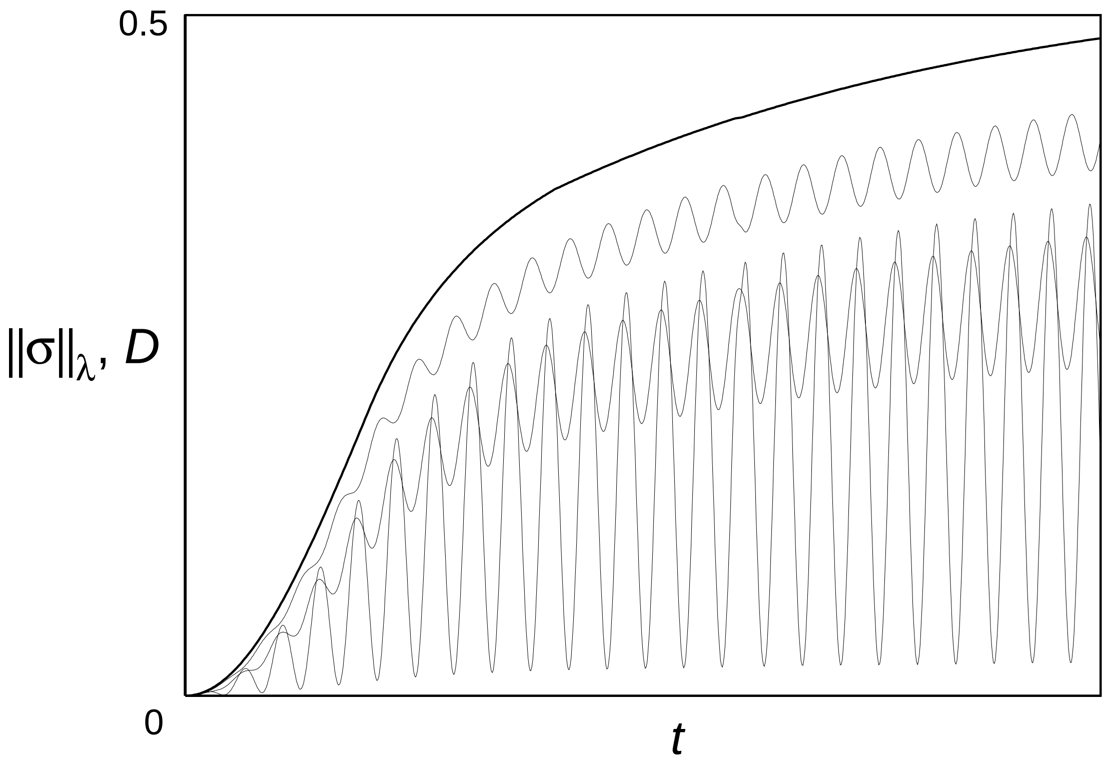
The Maximal Norm and Its Properties

Averaging over the initial density matrices smoothes out time-dependence at the frequencies of the system, leaving only the relaxation temporal dynamics.

Spin-1/2 interacting with an Ohmic bath of bosonic modes, in the short-time approximation.



Qubit Decoherence: Examples of Calculations

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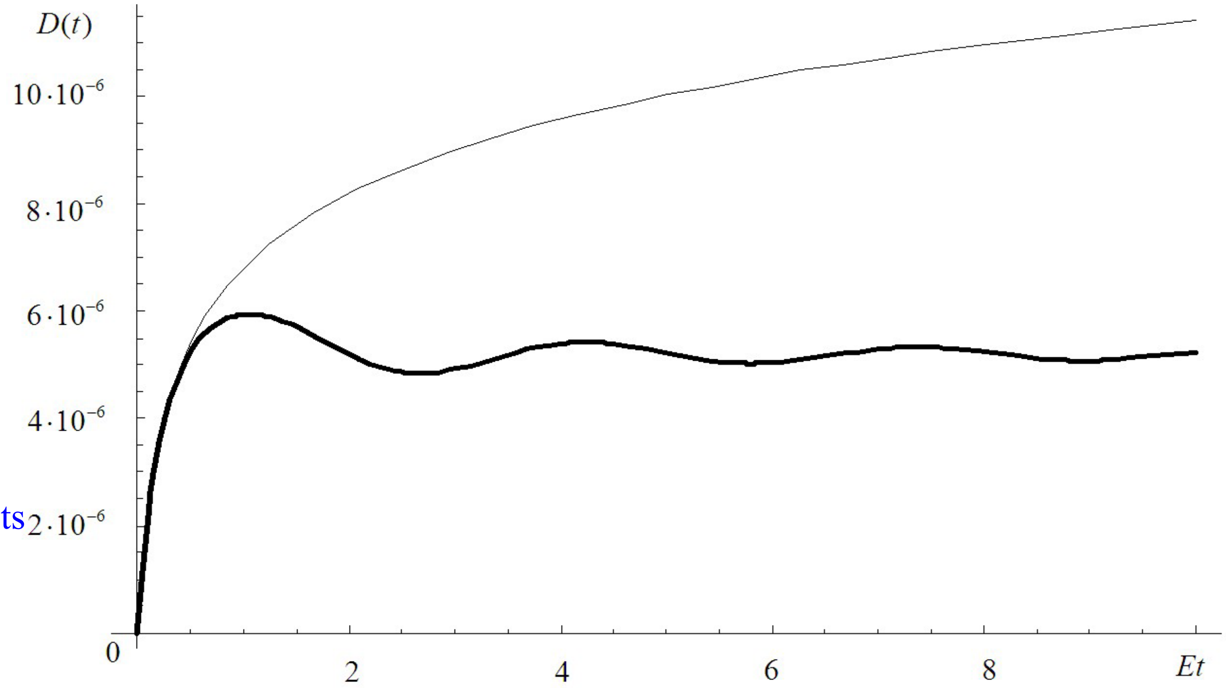
Adiabatic Approximation vs. Loss-DiVincenzo Approximation

D. Solenov and
V. Privman,
cond-mat/0403510,
Proc. SPIE (in print)



The LD (bottom) & AD (top) results for intermediate times and beyond, for $\omega_c/E = 30$ and $\alpha = 10^{-6}$.

Qubit Decoherence: Examples of Calculations

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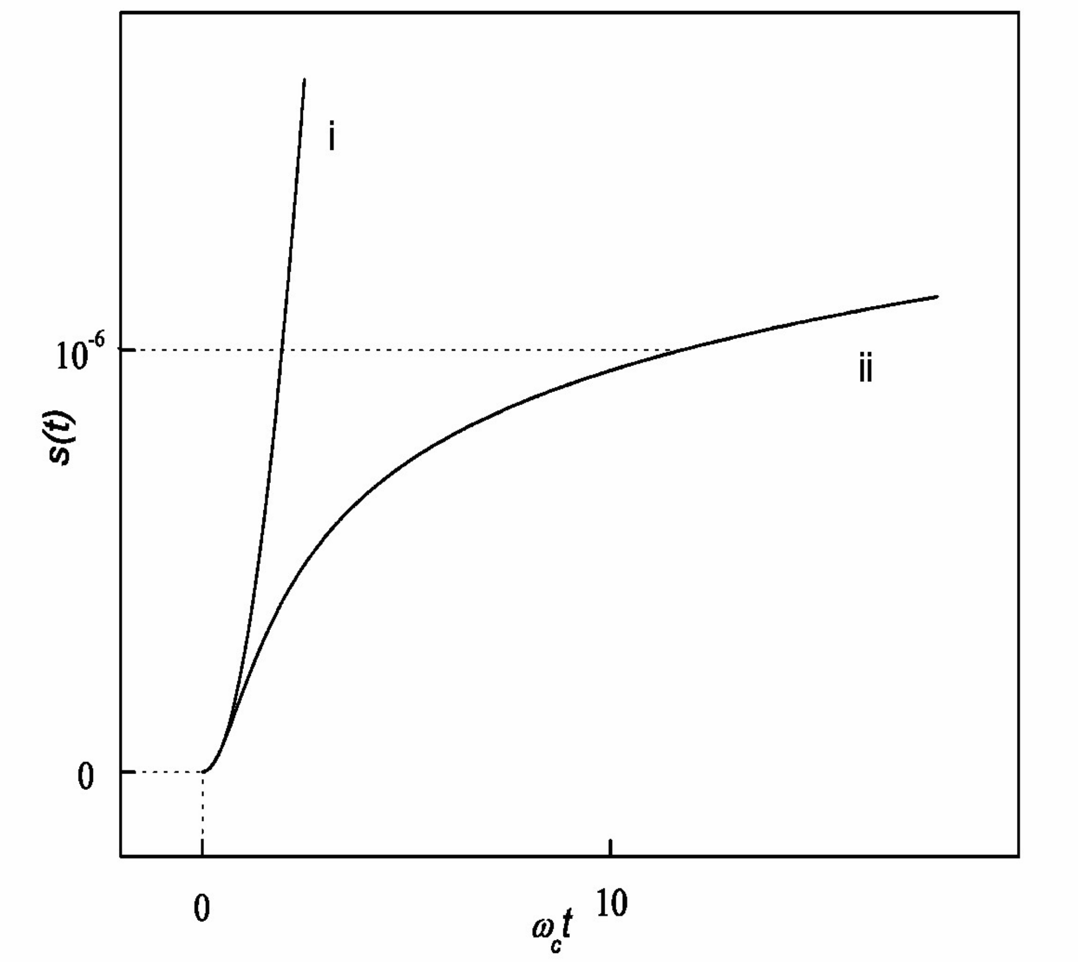
Models with Interactions of the form $\Lambda_S P_B^\dagger + \Lambda_S^\dagger P_B$

D. Tolkunov and
V. Privman,
cond-mat/0403348



Comparison between the $O(t^2)$ expansion, (i), and the short-time approximation, (ii), for a Jaynes-Cummings type model, for the idempotency defect measure.