Electron spin decoherence due to interaction with a nuclear spin bath

Center for Quantum Device Technology
Clarkson University

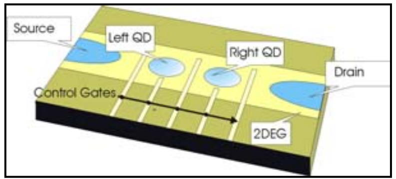
Presenter: Dr. Semion Saikin
email: saikin@clarkson.edu

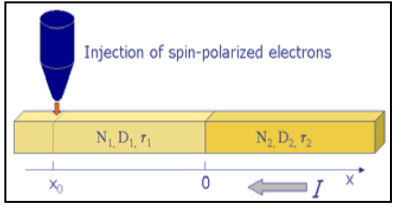
NSF-DMR-0121146, ITR/SY: Center for Modeling of Quantum Dynamics, Relaxation and Decoherence in Solid-State Physics for Information-Technology Applications

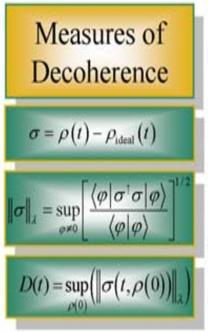
PI: Vladimir Privman, **Institution:** Clarkson University

Research Objectives

The main objectives of our program have been to explore coherent quantum mechanical processes in novel solid-state semiconductor information processing devices with components of atomic dimensions. These include quantum computers, spintronic devices, and nanometer-scale logic gates.



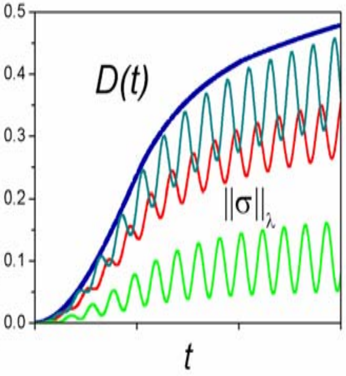



Measures of Decoherence

$$\sigma = \rho(t) - \rho_{\text{ideal}}(t)$$

$$\|\sigma\|_\lambda = \sup_{\varphi \neq 0} \left[\frac{\langle \varphi | \sigma^\dagger \sigma | \varphi \rangle}{\langle \varphi | \varphi \rangle} \right]^{1/2}$$

$$D(t) = \sup_{\rho(0)} \left(\| \sigma(t, \rho(0)) \|_\lambda \right)$$



Significant Results

Our achievements to date include:

- ✓ new measures of initial decoherence, and evaluation of decoherence for spins in semiconductors;
- ✓ evaluation of solid-state quantum computing designs;
- ✓ studies of transport associated with quantum measurement;
- ✓ investigation of spin-polarized devices and role of nuclear spins in spintronics and quantum computing;
- ✓ general contributions to quantum computing algorithms and to time-dependent and phase-related properties of open many-body quantum mechanical systems;
- ✓ novel analytical and numerical Monte Carlo approaches to studying spin-polarization control for spintronic device modeling;
- ✓ investigation of spin relaxation dynamics in two-dimensional semiconductor heterostructures.

Approach

Our approach has been truly interdisciplinary. For example, in developing new measures of decoherence for quantum computing, we have employed concepts from many-body quantum physics, computer error-correction algorithms, and nonequilibrium statistical mechanics. In our description of spintronic devices, we have utilized large-scale Monte Carlo simulations, knowledge from solid-state physics of semiconductors and from the microelectronics area of electrical engineering, as well as novel ideas of coherent control of quantum dynamics.

Our approach has been to design and evaluate architectures that allow implementation of many gate cycles during the relaxation and decoherence times. This requires development of techniques to evaluate all the relevant time scales: single- and two-quantum-bit gate “clock” times, as well as time scales of relaxation processes owing to the quantum bit (e.g., spin) interactions with environment, such as phonons or surrounding spins. We have also studied spin-control and charge carrier transport for spintronics and quantum measurement.

Broader Impact

We have extensive research *collaborations* with leading experimental and theoretical groups. The *educational impact* has included training undergraduate and graduate students, postdoctoral researchers, and development of three new courses to introduce quantum device and quantum algorithmic concepts to graduate and undergraduate students. Our program has contributed to *homeland security* and received funding from the National Security Agency.

Our outreach program has included sponsoring presentation events, and an international workshop series *Quantum Device Technology*, held in May of 2002 and May 2004, and sponsored by the Nanotechnology Council of IEEE and NSA (via ARO). We have worked with the REU site for students at SUNY Potsdam to guide several *undergraduate research projects* in the topics of quantum computing and quantum algorithms.

Collaboration

Experiment:

Sean Barrett, Yale

Hong Wen Jiang, UCLA

Marco Fanciulli, MDM Laboratory, Milan, Italy

Theory:

Leonid Fedichkin, Clarkson University

Boris Malkin, Kazan State University, Russia

Dima Mozyrsky, LANL

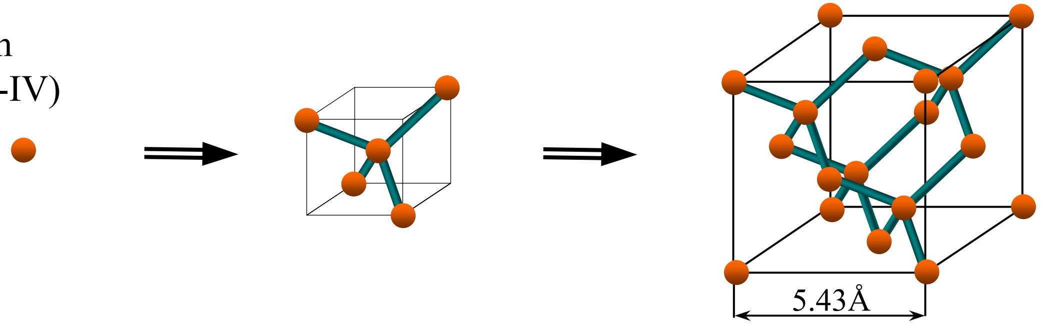
Vladimir Privman, Clarkson University

Israel Vagner, Holon Institute of technology, Israel

Donor electron spin in Si:P

Structure

Si atom
(group-IV)



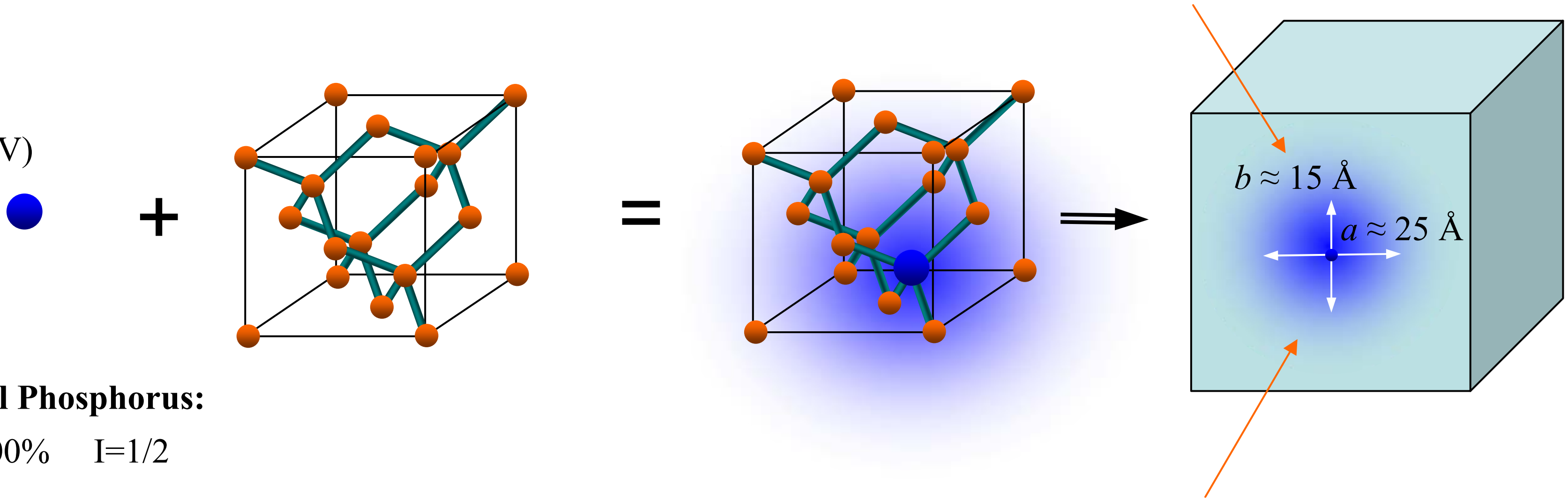
Diamond crystal structure

Natural Silicon:

^{28}Si – 92%
^{29}Si – 4.7% I=1/2
^{30}Si – 3.1%

^{31}P electron spin (T=4.2K)
T_1~ min T_2~ msecs

P atom
(group-V)



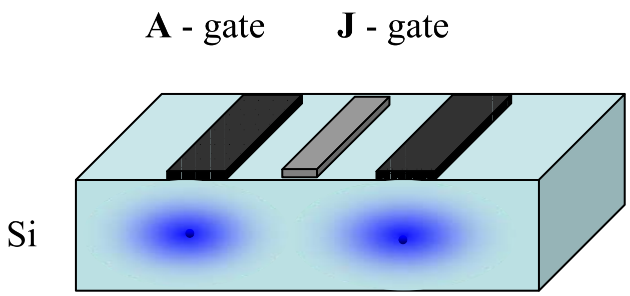
Natural Phosphorus:

^{31}P – 100% I=1/2

In the effective mass approximation the electron wave function is *s*-like: $F(\mathbf{r}) = \frac{1}{\sqrt{\pi ab}} e^{-\sqrt{(x^2+y^2)/a^2+z^2/b^2}}$

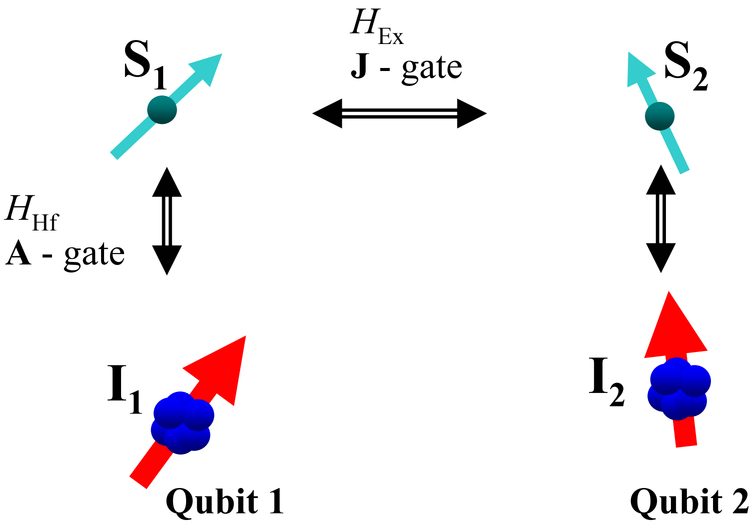
Donor electron spin in Si:P

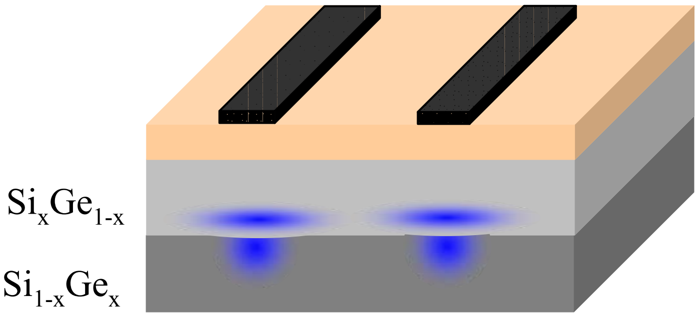
An application for QC



B.E.Kane, Nature **393** 133 (1998)

^{31}P donor
Qubit – nuclear spin
Qubit-qubit interaction – electron spin



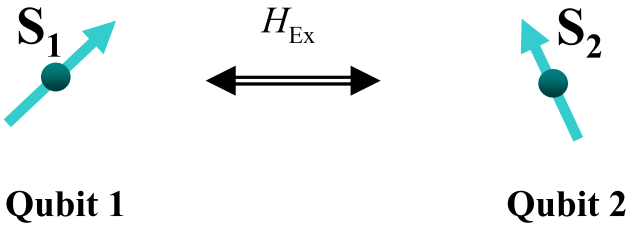


Bohr Radius:
Si: $a \approx 25$ Å
$b \approx 15$ Å

Ge: $a \approx 64$ Å
$b \approx 24$ Å

R.Vrijen, E.Yablonovitch, K.Wang, H.W.Jiang, A.Balandin, V.Roychowdhury, T.Mor, D.DiVincenzo, Phys. Rev. A **62**, 012306 (2000)

^{31}P donor
Qubit – electron spin
Qubit-qubit interaction – electron spin



Donor electron spin in Si:P

Sources of decoherence

- **Interaction with phonons**

  D. Mozyrsky, Sh. Kogan, V. N. Gorshkov, G. P. Berman
  Phys. Rev. B **65**, 245213 (2002)

- **Gate errors**

  X. Hu, S. Das Sarma, cond-mat/0207457

- **Interaction with ^{29}Si nuclear spins**

  Theory

  I. A. Merkulov, Al. L. Efros, M. Rosen, Phys. Rev. B 65, 205309 (2002)

  S. Saikin, D. Mozyrsky, V. Privman, Nano Letters 2, 651 (2002)

  R. De Sousa, S. Das Sarma, Phys. Rev. B 68, 115322 (2003)

  S. Saikin, L. Fedichkin, Phys. Rev. B 67, 161302(R) (2003)

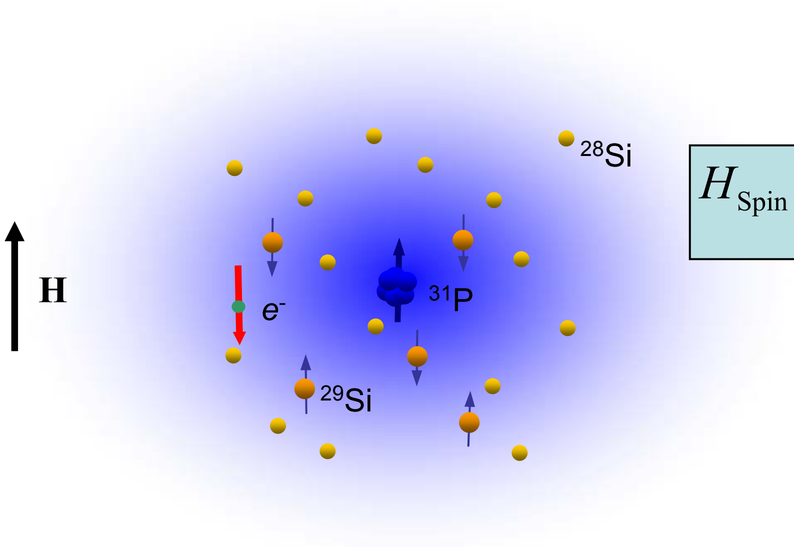
  J. Schliemann, A. Khaetskii, D. Loss, J. Phys., Condens. Matter 15, R1809 (2003)

  Experiments

  A. M. Tyryshkin, S. A. Lyon, A. V. Astashkin, A. M. Raitsimring, Phys. Rev. B 68, 193207 (2003)
  M. Fanciulli, P. Hofer, A. Ponti, Physica B 340–342, 895 (2003)
  E. Abe, K. M. Itoh, J. Isoya S. Yamasaki, cond-mat/0402152 (2004)

Donor electron spin in Si:P

Spin Hamiltonian



$$H_{\mathrm{Spin}} = \underbrace{H_Z^e + \sum_i H_Z^{\mathrm{nucl}}(i)}_{\text{Effect of external field}} + \underbrace{\sum_i H_{\mathrm{Hf}}(i)}_{\text{Electron-nuclei interaction}} + \underbrace{\sum_{i\neq j} H_{\mathrm{Dip}}(i,j)}_{\text{Nuclei-nuclei interaction}}$$

Effective Bohr radius ~ 20-25 Å
Lattice constant = 5.43 Å

In a natural Si crystal the donor electron interacts with ~ 80 nuclei of ^{29}Si

System of ^{29}Si nuclear spins can be considered as a spin bath

Electron spin Zeeman term: $H_Z^e = g\beta \mathbf{HS}$

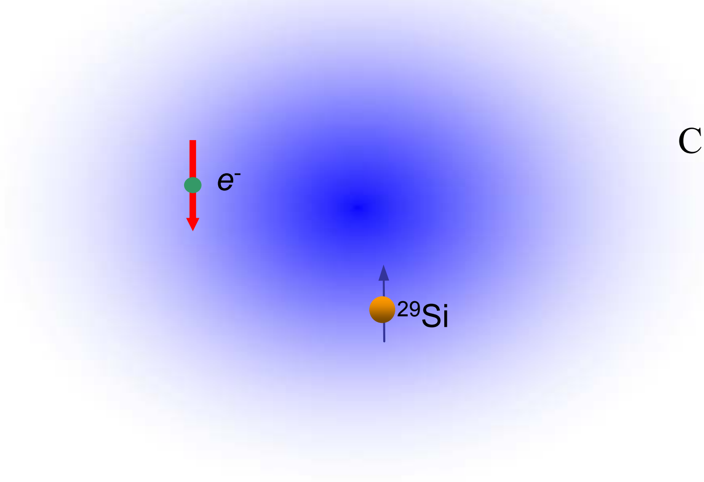
Nuclear spin Zeeman term: $H_Z^{\mathrm{nulc}}(i) = -\gamma_i \hbar \mathbf{HI}$

Hyperfine electron-nuclear spin interaction: $H_{\mathrm{Hf}}(i) = \mathbf{S}\mathbf{A}_i\mathbf{I}^i$

Dipole-dipole nuclear spin interaction: $H_{\mathrm{Dip}}(i,j) = \mathbf{I}^i\mathbf{D}_{ij}\mathbf{I}^j$

Donor electron spin in Si:P

Hyperfine interaction



Contact interaction:

$$H_{\text{Cont}} = A\mathbf{S}\mathbf{I}$$

Dipole-dipole interaction:

$$H_{\text{Dip}} = \frac{\boldsymbol{\mu}_e\boldsymbol{\mu}_n}{r^3} - \frac{3(\boldsymbol{\mu}_e\mathbf{r})(\boldsymbol{\mu}_n\mathbf{r})}{r^5}$$

$$\Downarrow$$

Hyperfine interaction:

$$H_{\text{Hf}} = \begin{pmatrix} S_x & S_y & S_z \end{pmatrix} \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} \begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix}$$

Approximations:

Contact interaction only:

$$\mathbf{A} = \begin{pmatrix} A_{xx} & 0 & 0 \\ 0 & A_{yy} & 0 \\ 0 & 0 & A_{zz} \end{pmatrix}$$

High magnetic field

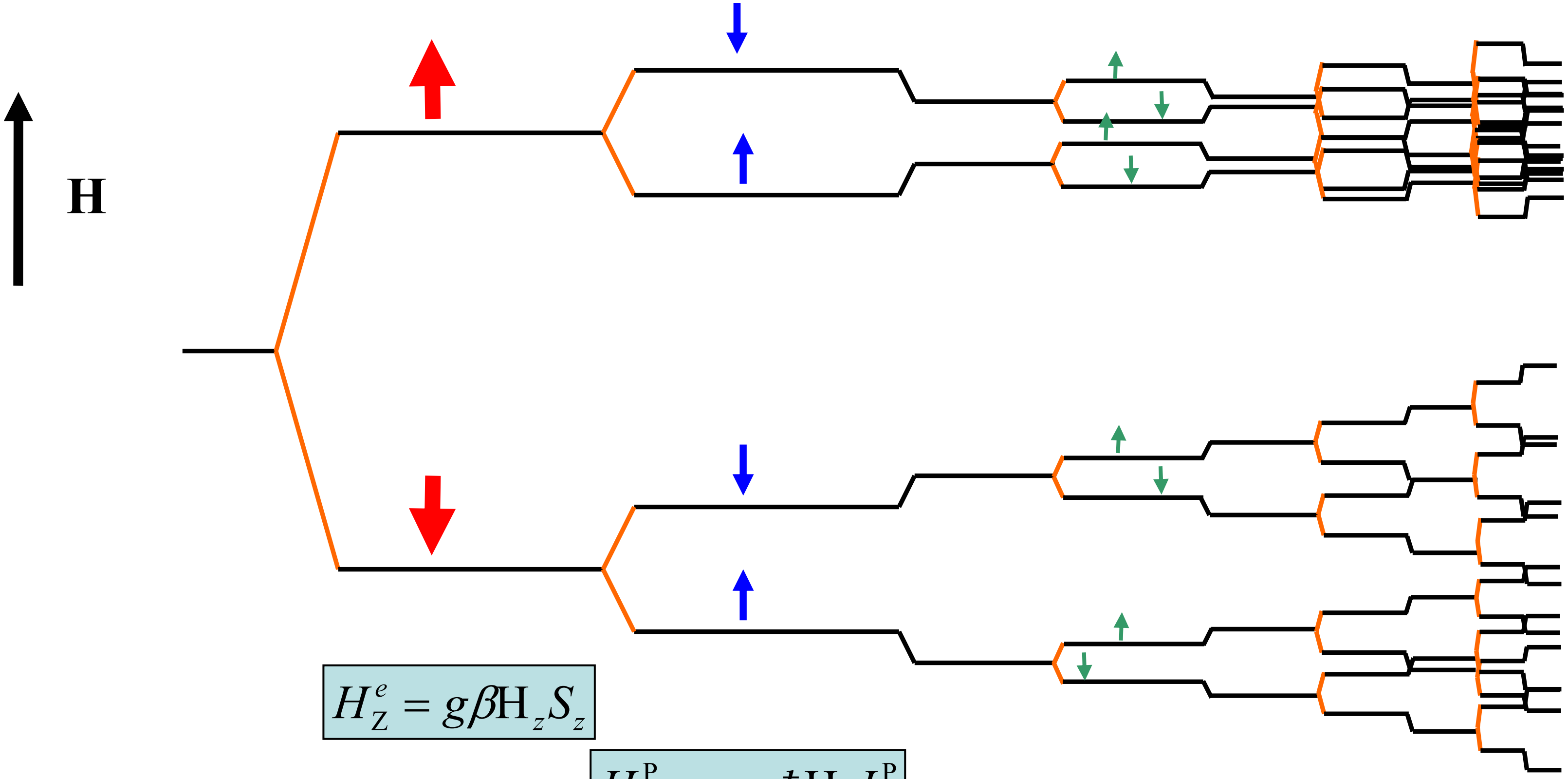
$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix}$$

Contact interaction
High magnetic field

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_{zz} \end{pmatrix}$$

Donor electron spin in Si:P

Energy level structure (high magnetic field)



$$H_{\mathrm{Z}}^{e} = g\beta \mathrm{H}_z S_z$$

$$H_{\mathrm{Z}}^{\mathrm{P}} = -\gamma_{\mathrm{P}} \hbar \mathrm{H}_z I_z^{\mathrm{P}}$$

$$H_{\mathrm{Hf}}^{\mathrm{P}} = A_{zz}^{\mathrm{P}} S_z I_z^{\mathrm{P}}$$

$$H_{\mathrm{Z}}^{\mathrm{Si}_1}$$

$$H_{\mathrm{Hf}}^{\mathrm{Si}_1}$$

…

↑ - ^{31}P electron spin

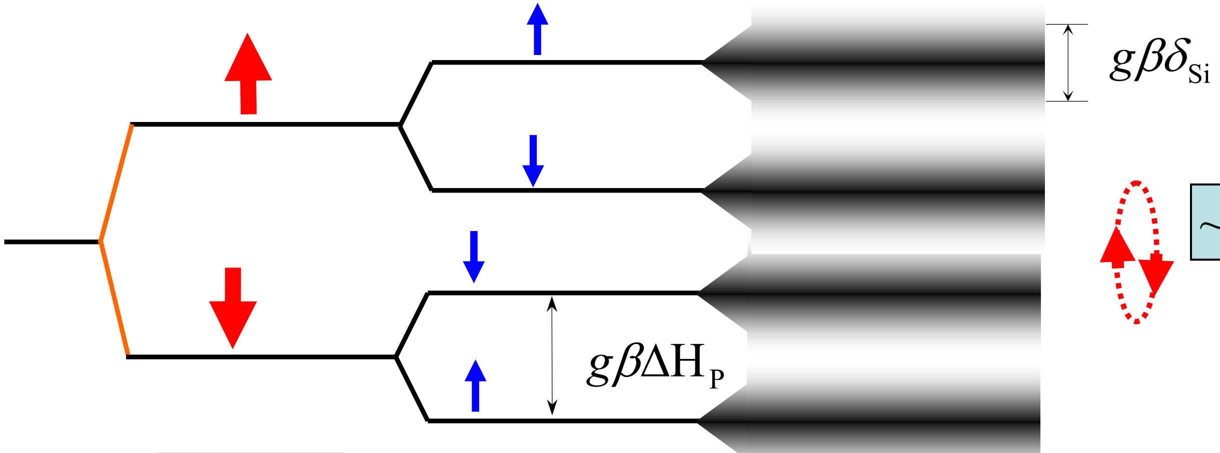
↑ - ^{31}P nuclear spin

↑ - ^{29}Si nuclear spin

Donor electron spin in Si:P

Effects of nuclear spin bath (low field)

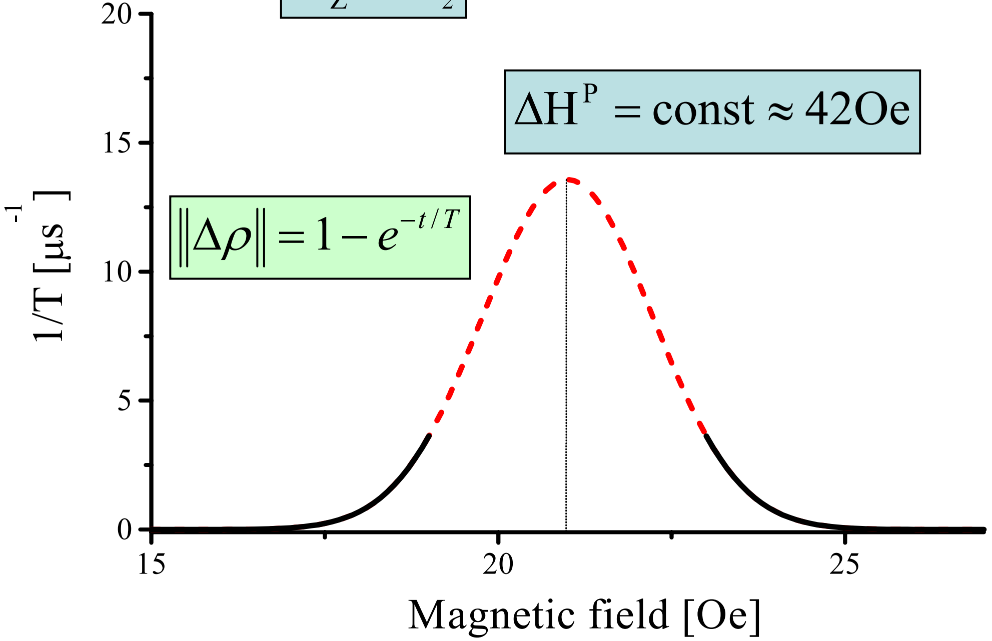
$$\mathbf{A} = \begin{pmatrix} A_{xx} & 0 & 0 \\ 0 & A_{yy} & 0 \\ 0 & 0 & A_{zz} \end{pmatrix}$$



$\sim (S_+ I_-^{\mathrm{Si}} + S_- I_+^{\mathrm{Si}})$

$H_{\mathrm{Z}}^{e} \sim \mathrm{H}_z$

$\delta_{\mathrm{Si}} = \mathrm{const} \approx 3\mathrm{Oe}$



$\Delta\mathrm{H}^{\mathrm{P}} = \mathrm{const} \approx 42\mathrm{Oe}$

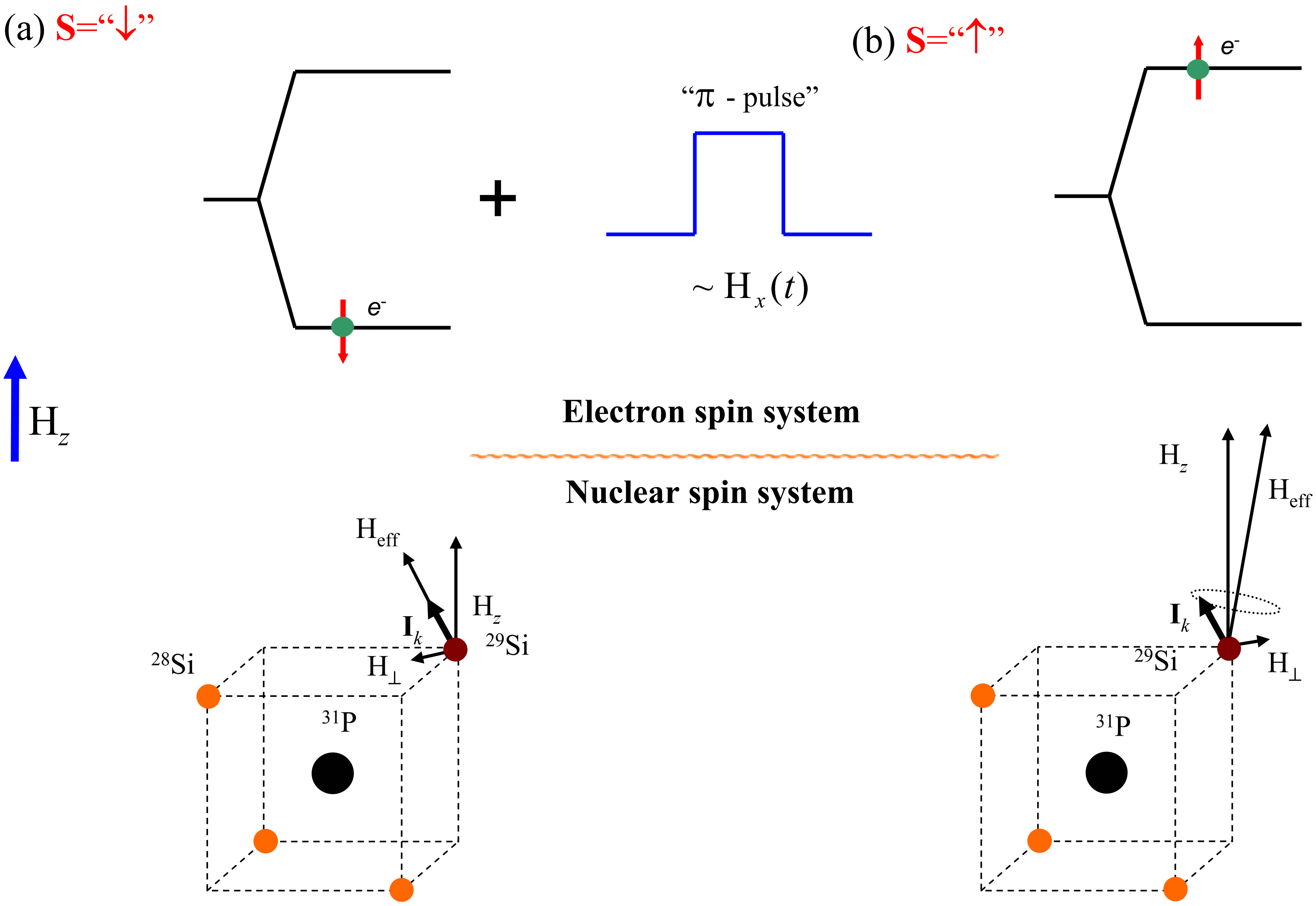
$\|\Delta\rho\| = 1 - e^{-t/T}$

S. Saikin, D. Mozyrsiky and V. Privman,
Nano Lett. **2**, 651-655 (2002)

Donor electron spin in Si:P

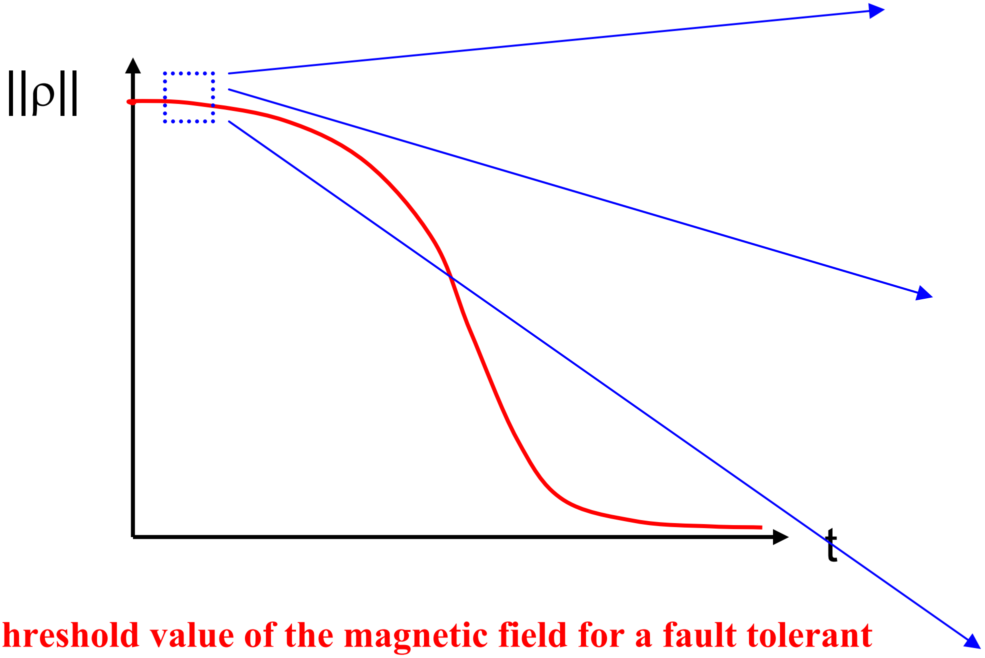
Effects of nuclear spin bath (high field)

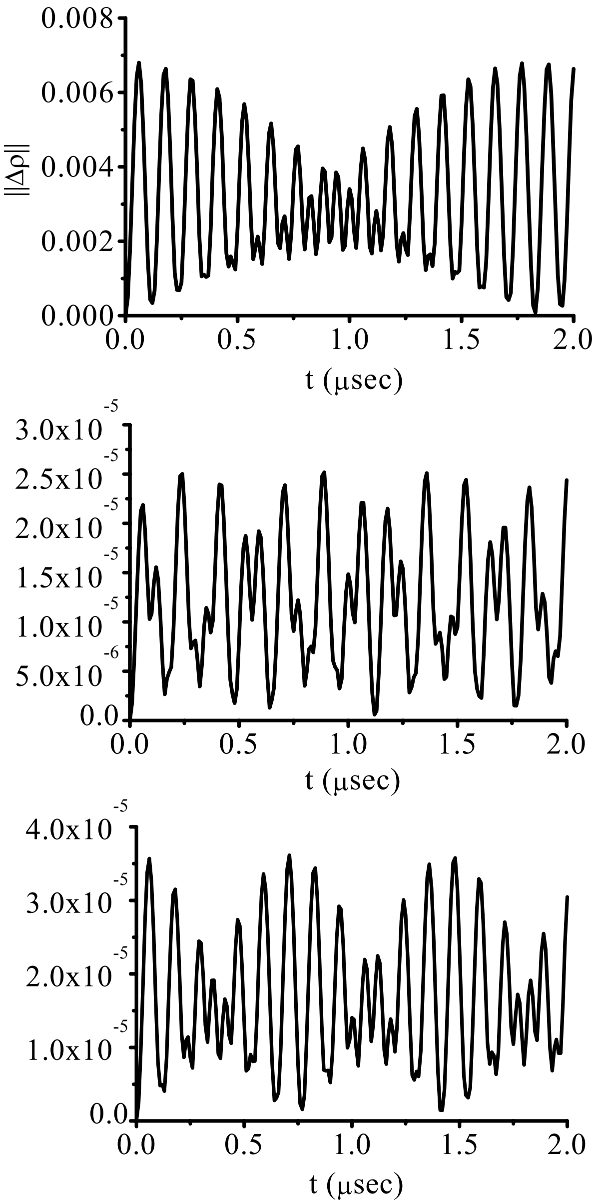
$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix}$$



Donor electron spin in Si:P

Hyperfine modulations of an electron spin qubit





Threshold value of the magnetic field for a fault tolerant ^{31}P electron spin qubit:

$$\|\Delta\rho_{\max}\| \sim \mathrm{H}^{-2}$$

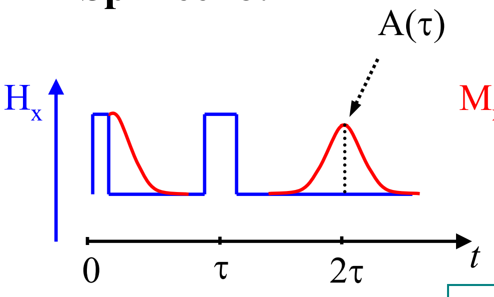
$$\mathrm{H}_{\mathrm{th}} \approx 9\mathrm{T}$$

S. Saikin and L. Fedichkin,
Phys. Rev. B **67**, article 161302(R), 1-4 (2003)

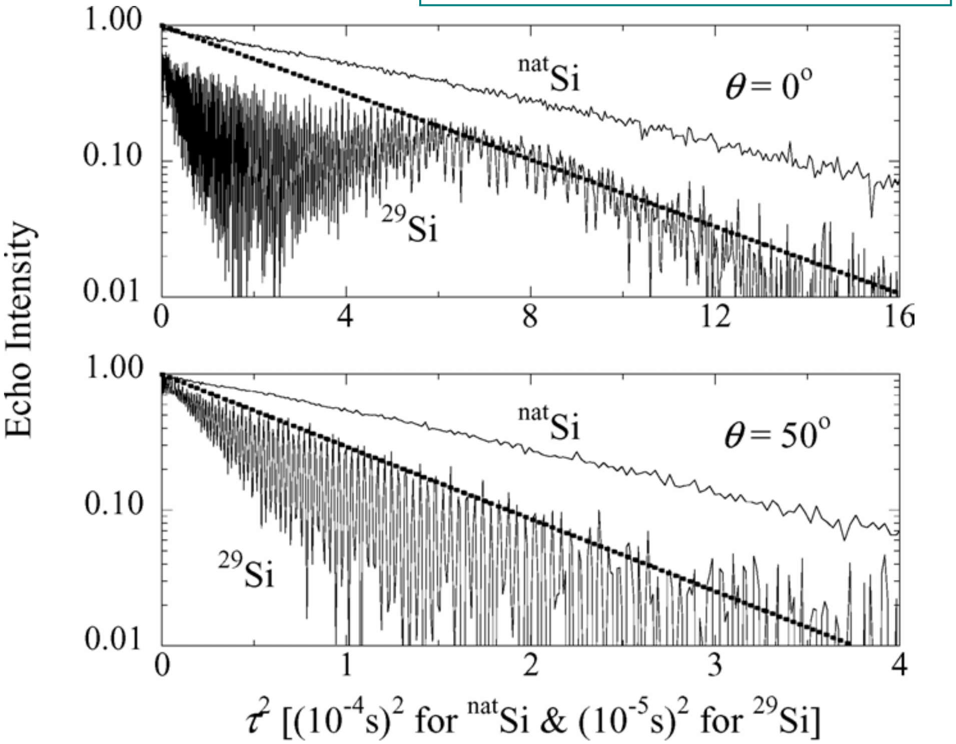
Donor electron spin in Si:P

Spin echo modulations: Experiment

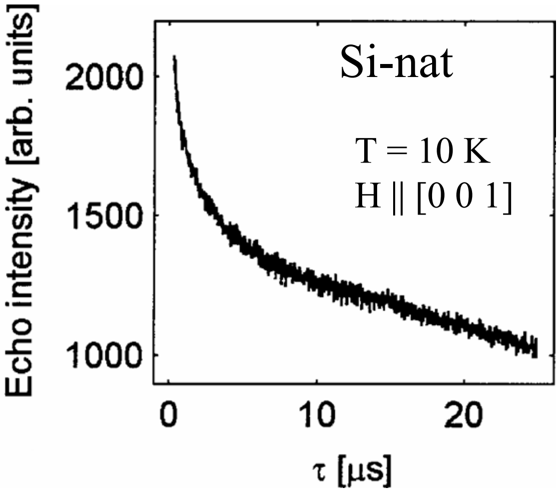
Spin echo:

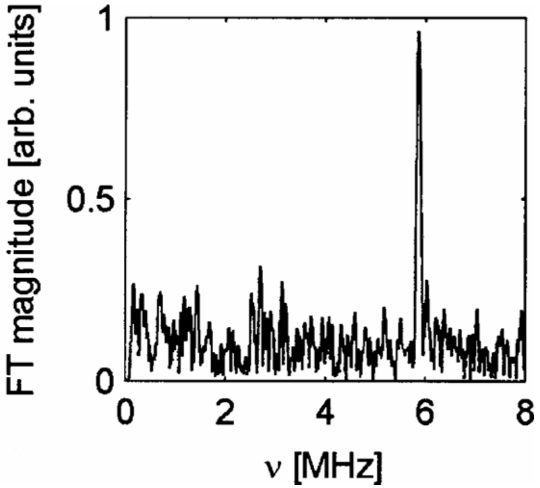


E. Abe, K. M. Itoh, J. Isoya
S. Yamasaki, cond-mat/0402152



M. Fanciulli, P. Hofer, A. Ponti
Physica B **340–342**, 895 (2003)





Conclusions

- Effects of a nuclear spin bath on the decoherence of an electron spin qubit in a Si:P system has been studied.

- A new measure of decoherence processes has been applied.

- In the low field regime the coherence of a qubit decays exponentially with a characteristic time, $T \sim 0.1\ \mu\text{sec}$.

- In the high magnetic field regime, quantum operations cause the qubit state to deviate from the ideal state. The characteristic time for these processes is on the order of 0.1 μsec.

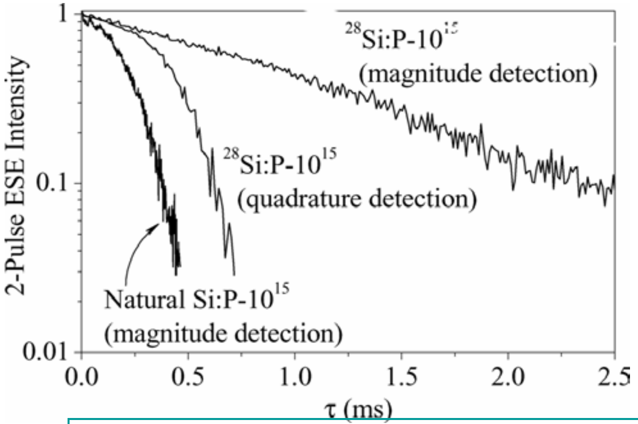
- The threshold value of an external magnetic field required for fault-tolerant quantum computation is $H_{ext} \sim 9$ Tesla.

S. Saikin, D. Mozyrsky, V. Privman, Nano Letters 2, 651 (2002)

S. Saikin, L. Fedichkin, Phys. Rev. B 67, 161302(R) (2003)

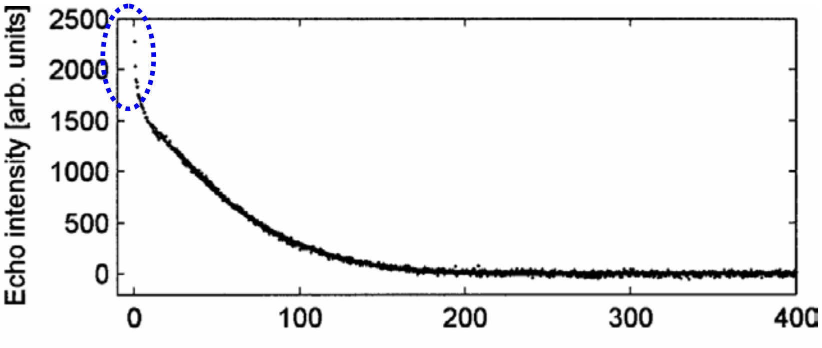
Future prospects

- **Spin diffusion**



A. M. Tyryshkin, S. A. Lyon,
A. V. Astashkin, and A. M. Raitsimring
Phys. Rev. B **68**, 193207 (2003)

- **Initial drop of spin coherence**



M. Fanciulli, P. Hofer, A. Ponti
Physica B **340–342**, 895 (2003)

- **Control for spin-spin coupling in solids**

S. Barrett's Group, Yale

M. Fanciulli Group, MDM Laboratory, Italy

Development of error avoiding methods for spin qubits in solids.