Breaking Time Translation Symmetry in Quantum Systems

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1508.03344

2. C. W. von Keyserlingk, V. Khemani e S. L. Sondhi,

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1612.08758

In 2012 Wilczek asked

Can a quantum system spontaneously break time translation symmetry (TTS)?

and called the hypothesized TTSB state a <u>time crystal</u>

By 2015 various people, most notably Watanabe & Oshikawa, argued that time crystals did not exist.

In 2016 two experimental groups - Monroe & Lukin - reported observations of time crystals.

Standard Lore on SSB

Internal Symmetries (Abelian)

$g \in G$ represented by $W(g) = \bigotimes_i W_i(g)$

G is broken to H.

$W^\dagger(g)\, \phi_{i,\alpha}\, W(g) = e^{i\theta_{g,\alpha}}\, \phi_{i,\alpha}$ irrep

$W^\dagger(g)\, |n\rangle = e^{i\lambda_{g,n}}\, |n\rangle$ as $[H, W(g)] = 0$

$\Rightarrow \quad \langle n|\, \phi_{i,\alpha}\, |n\rangle = 0$

SSB if $\lim_{|i-j|\to\infty} \lim_{L\to\infty} |\langle n|\, \phi_{i,\alpha}\, \phi_{j,\bar\alpha}\, |n\rangle| \neq 0$

for ϕ_α trivial under H but non-trivial under G; LOP

Example

$$H = J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + h \sum_i \sigma_i^x$$

$$G = \{ \mathbb{1}, P_x \} \equiv \mathbb{Z}_2$$

$$P_x = \prod_i \sigma_i^x$$

$$\phi_i = \sigma_i^z$$

$P_x \phi_i P_x = -\phi_i$ while $P_x H P_x = H$

G broken if $\lim \langle n | \sigma_i^z \sigma_j^z | n \rangle \neq 0$

Spatial Symmetries (Translation)

$$x \xrightarrow{T(x)} x+r$$

Order parameter is sum of local terms

$$O_k = \frac{1}{L} \sum_r e^{ikr} \, O(r) \qquad \text{SLOP}$$

Symmetry is <u>not</u> represented by finite depth unitary circuit

(Won't use this option today)

What about TTS?

Time independent Hamiltonian Systems

Translation by $t \longrightarrow U(t) = e^{+iHt}$ $\qquad G \equiv \mathbb{R}$

$[H, U(t)] = 0 \quad \forall t$

$U(t)\,|n\rangle = e^{-iE_n t}\,|n\rangle$

Order parameter?

$O_{nm} = |n\rangle\langle m|$

$U^\dagger(t)\, O_{nm}\, U(t) = e^{i(E_n - E_m)t}\, O_{nm} \equiv e^{i\Delta_{m,n}}\, O_{nm}$

Can one take linear combinations of the O_{nm} to get LOP?

Floquet systems

$H(t+T) = H(t)$ $\qquad G \equiv \mathbb{Z}$

$U(T)|n\rangle = e^{-i\epsilon_n T}|n\rangle$

$U(T)^\dagger O_{nm} U(T) = e^{i(\epsilon_n-\epsilon_m)T} O_{n,m} \equiv e^{i\Delta_{nm}} O_{nm}$

Same question.

More precisely, want $\phi_{i,\alpha}$ such that

$U^\dagger(t)\,\phi_{i,\alpha}\,U(t) = e^{i\Delta_\alpha t}\,\phi_{i,\alpha}$

$\langle n|\phi_{i,\alpha}|n\rangle = 0$

$\lim_{|i-j|\to\infty}\lim_{L\to\infty} |\langle n|\phi_{i,\alpha}\phi_{j,\bar\alpha}|n\rangle| = c_0 \neq 0$ $\qquad$ <u>TTSB</u>

for $\mathbb{R}/\mathbb{Z}$ broken to a ~~subset~~/subgroup.

When can we observe TTSB?

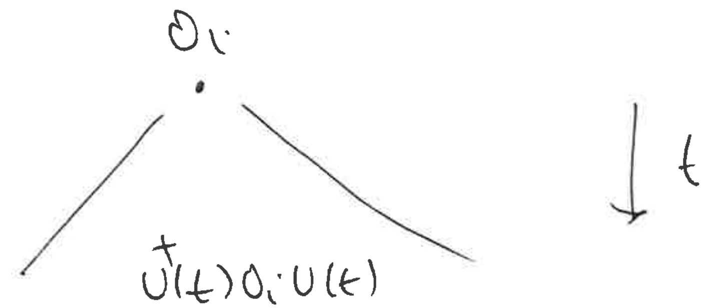
Absent for thermalizing systems

Hamiltonian $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ T

$\qquad\qquad\qquad$ eigenstates obey ETH $\quad$ with

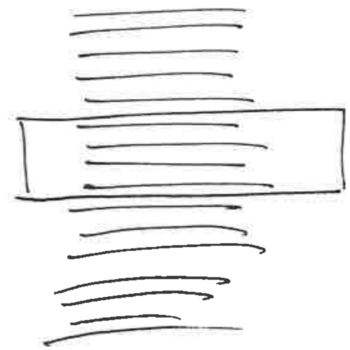
Floquet $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ $T=\infty$

For such systems local operators spread ballistically with time



Many body localized systems?

ETH breaks down



μcanonical window

$\langle n|O|n\rangle \neq \langle O\rangle_{\mu\text{canonical}}$

due to large fluctuations between eigenstates. Allows eigenstate order.

Huse, Nandkishore, Oganesyan, Pal & Sondhi (2013)

__But__ for time independent H, local operators still spread in time, albeit slowly.

That leaves Floquet-MBL and we will succeed there!

Binary Ising Drives

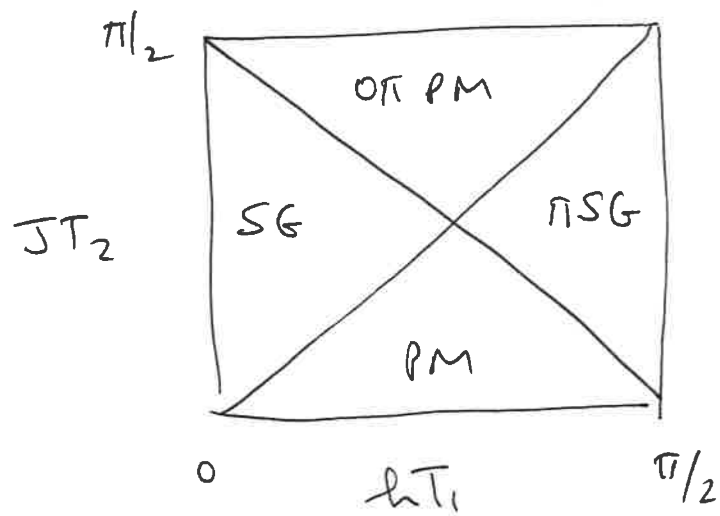
Khemani et al

$$H(t) = \begin{cases} H_1 = -h \sum_i \sigma_i^x & 0 < t < T_1 \\ H_2 = -J \sum_i \sigma_i^z \sigma_{i+1}^z & T_1 < t < T_1 + T_2 = T \end{cases}$$

Really $h_i = h + \delta h_i$

$J_i = J + \delta J_i$

Phase diagram



πSG breaks TTS (renamed FTC / DTC by Q / Berkeley)

Along $hT_1 = \frac{\pi}{2}$

$$U(T) = P_x \, e^{iJT_2 \sum_i \sigma_i^z \sigma_{i+1}^z}$$

$$U^\dagger(T) \sigma_i^z U(T) = -\sigma_i^z \qquad \left(P_x \sigma_i^z P_x = -\sigma_i^z \right)$$

$$U^\dagger(2T) \sigma_i^z U(2T) = +\sigma_i^z$$

<u>Eigenstates?</u> $|z_1 \dots z_N\rangle$ where $\sigma_i^z |z_1 \dots z_N\rangle = z_i \cdot |z_1 \dots z_N\rangle$

Then $|\pm\rangle = \frac{1}{\sqrt{2}} \left(|z_1 \dots z_N\rangle \pm |\bar{z}_1 \dots \bar{z}_N\rangle \right)$ are eigenstates

$$U(T) |\pm\rangle = (\pm 1) \, e^{iJT_2 \sum_i z_i z_{i+1}} |\pm\rangle$$

π - doublets

Break global Z_2 and TTS.

Observe that

ETH violated

Thermal average is trivial

Set of $\Delta_{mn} = \epsilon_n - \epsilon_m$ massively degenerate

All of these survive for arbitrary weak perturbations

$\delta H(t) = \delta H(t+T)$

Example: Add all three random fields

Need to prepare eigenstates? <u>No</u>

A generic starting state $|\psi(0)\rangle$ leads to a late time state $|\psi(t)\rangle$ that exhibits period doubling in expectation values of local observables.

Monroe et al   Ion traps - small Ising system

Lukin et al   NV centers, possibly critical