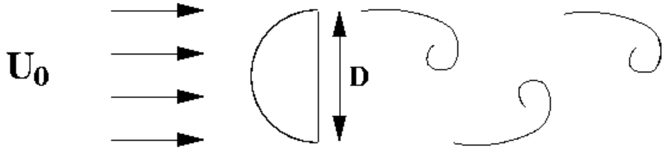
Liquid Flows

Non-dimensionalization

$$t_c = \frac{D}{U} \qquad t_d = \frac{D^2}{\nu}$$

convective *diffusive*



•Incompressible High-Speed Flows:

$$\frac{\partial \omega}{\partial t_c^*} + \nabla \cdot (u\omega) = (\omega \cdot \nabla) u + \mathrm{Re}^{-1} \nabla^2 \omega$$

$$\nabla \cdot u = 0$$

•Incompressible Low-Speed Flows:

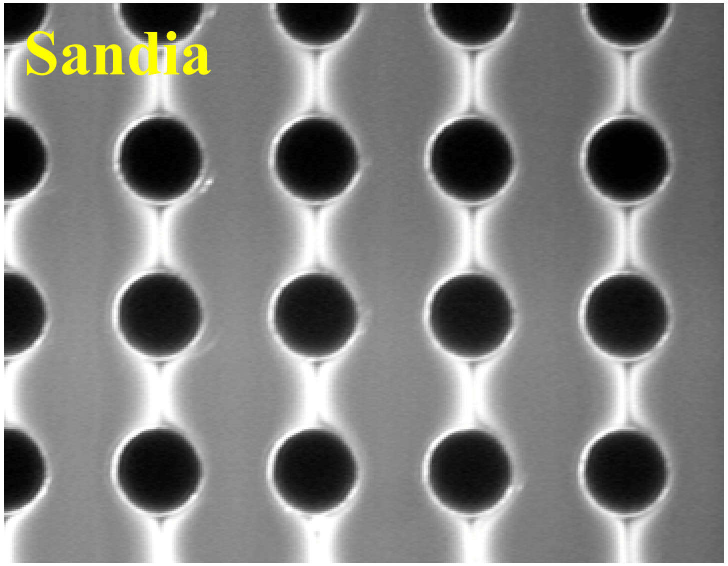
$$\frac{\partial \omega}{\partial t_d^*} + \mathrm{Re} \nabla \cdot (u\omega) = \mathrm{Re}(\omega \cdot \nabla) u + \nabla^2 \omega$$

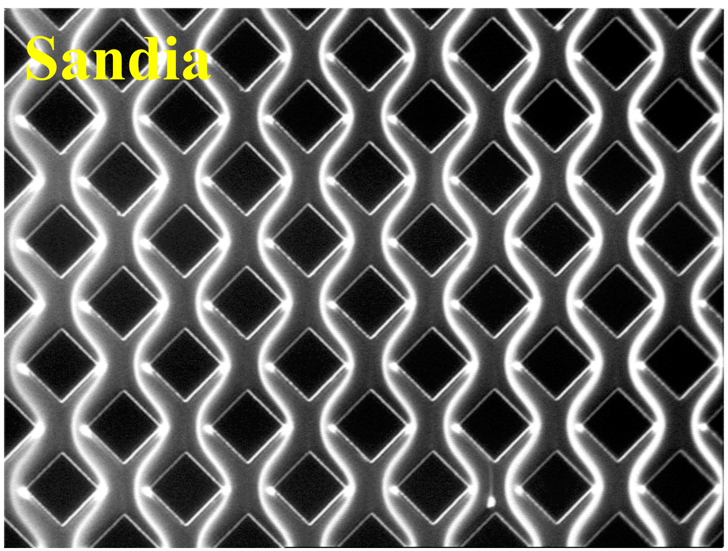
•Steady-state Stokes Flow:

$$-\mu \nabla^2 u + \nabla p = f$$

$$\nabla \cdot u = 0$$

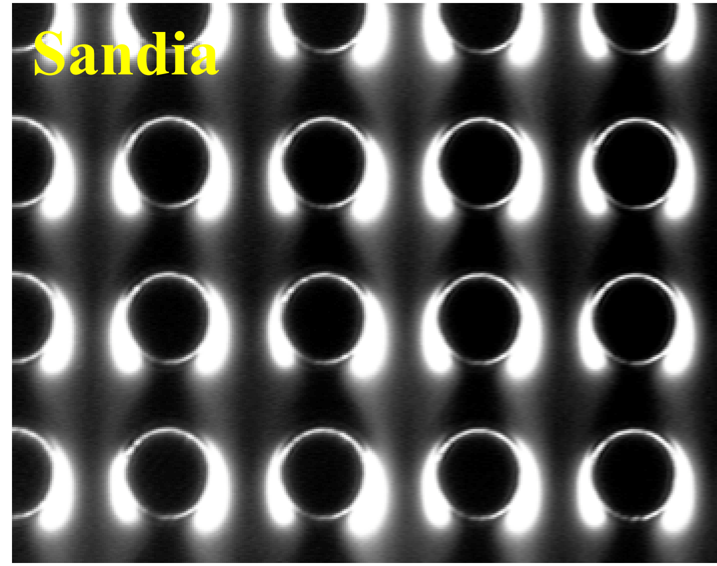
Filamentary Dielectrophoresis

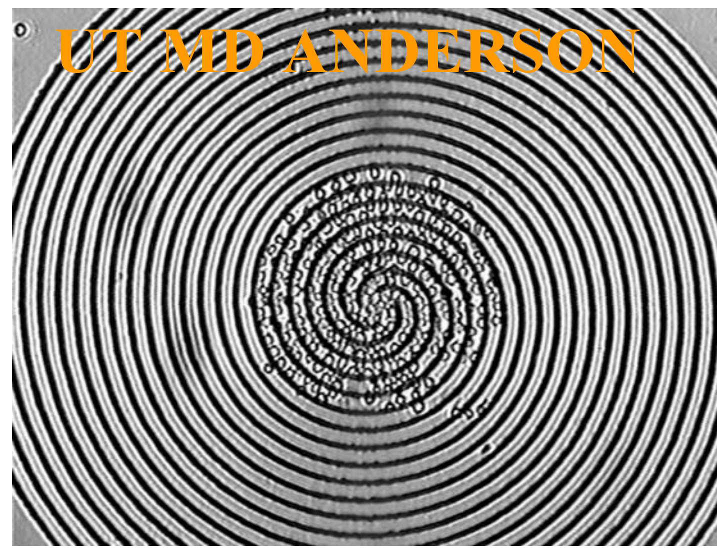




Courtesy of Eric Cummings,
Sandia National Laboratories

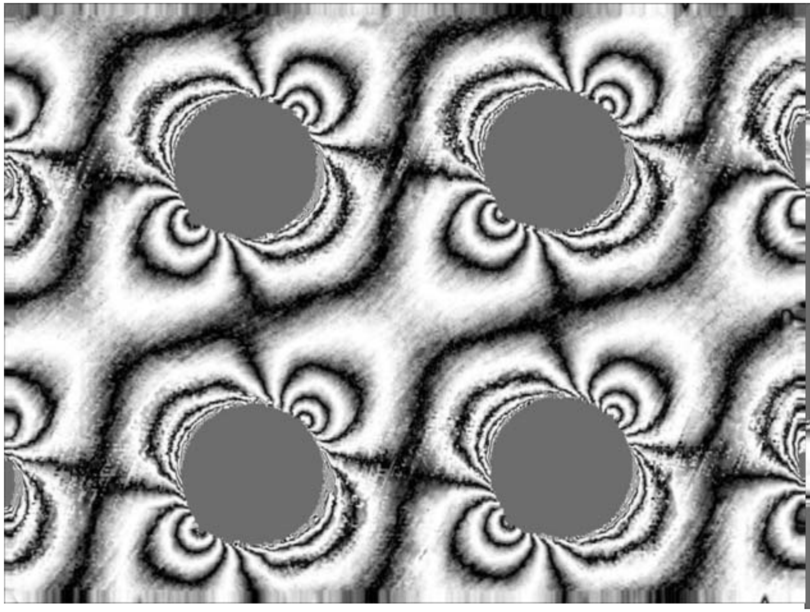
Trapping Dielectrophoresis



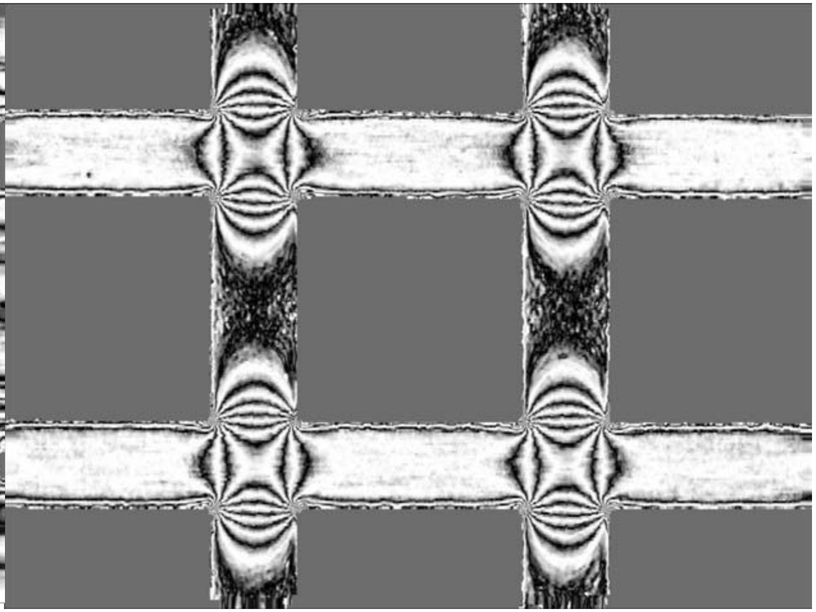


Courtesy of Peter Gascoyne,
UT MD Anderson

Electroosmotic Flow: Experimental Results



Speed contours in an array of posts at 45^o wrt the applied electric field 2V/mm



Streamwise velocity contours. Applied electric field 1V/mm from left to right

Courtesy of Eric Cummings, Sandia National Laboratories

Micro-PIV results presented in the form of simulated interferogram (Cummings, AIAA-01-1163)

Electro-Kinetic Effects

- **Electrophoresis**
  - Motion of charged surface relative to the stationary liquid by an applied electric field.
- **Electroosmosis**
  - Motion of ionized liquid relative to the stationary charged surface by applied electric field.
- **Streaming Pot.**
  - Electric field created by the motion of ionized fluid along stationary charged surfaces
- **Sedimentation Pot.**
  - Electric field created by the motion of charged particles relative to a stationary liquid

Other Microfluidic Particle Sorting/Separation Techniques

Isoelectric focusing is the migration of charged particles under pH gradients to a location in the buffer, where they have zero net charge

Dielectrophoresis is the motion of polarizable particles that are suspended in an electrolyte and subjected to a spatially non-uniform electric field. The particle motion is produced by the dipole moments induced on the particle and the suspending fluid.

Governing Equations for Electroosmosis

Poisson-Boltzmann Equation

$$\nabla^2 \psi = -\frac{4\pi\rho_e}{D}$$

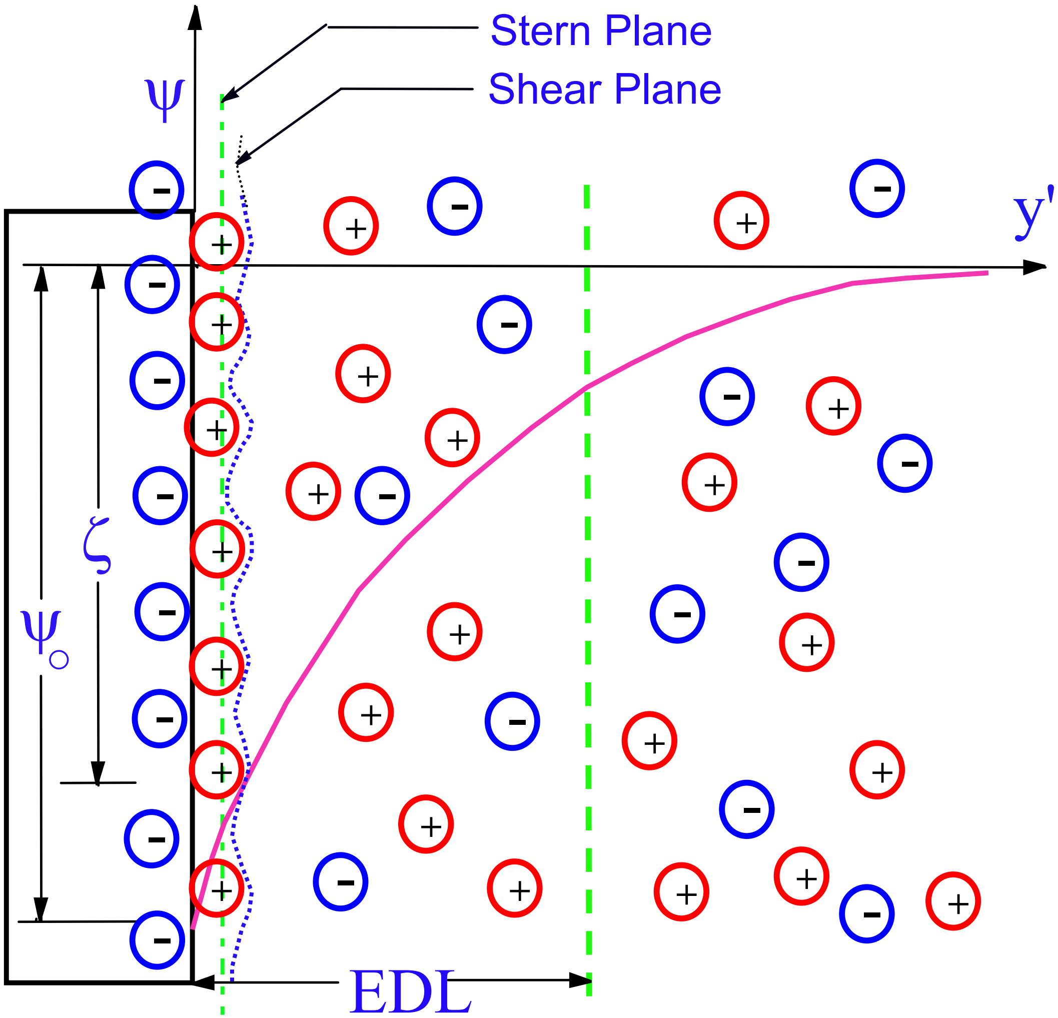
$$\rho_e = -2n_o ez \sinh(ez\psi / k_b T)$$

$$\alpha = ez\zeta / k_b T$$

$$\omega = \sqrt{\frac{8\pi n_o e^2 z^2}{D k_b T}}$$

$$\beta = (\omega h)^2 / \alpha$$

$$\nabla^2 \psi^* = \beta \sinh(\alpha \psi^*)$$



New Equations

$$\nabla^2 \psi = -\frac{4\pi\rho_e}{D}$$

$$\rho_e = -2n_o ez \sinh(ez\psi / k_b T)$$

$$\alpha = ez\zeta / k_b T$$

$$\omega = \sqrt{\frac{8\pi n_o e^2 z^2}{D k_b T}}$$

$$\beta = (\omega h)^2 / \alpha$$

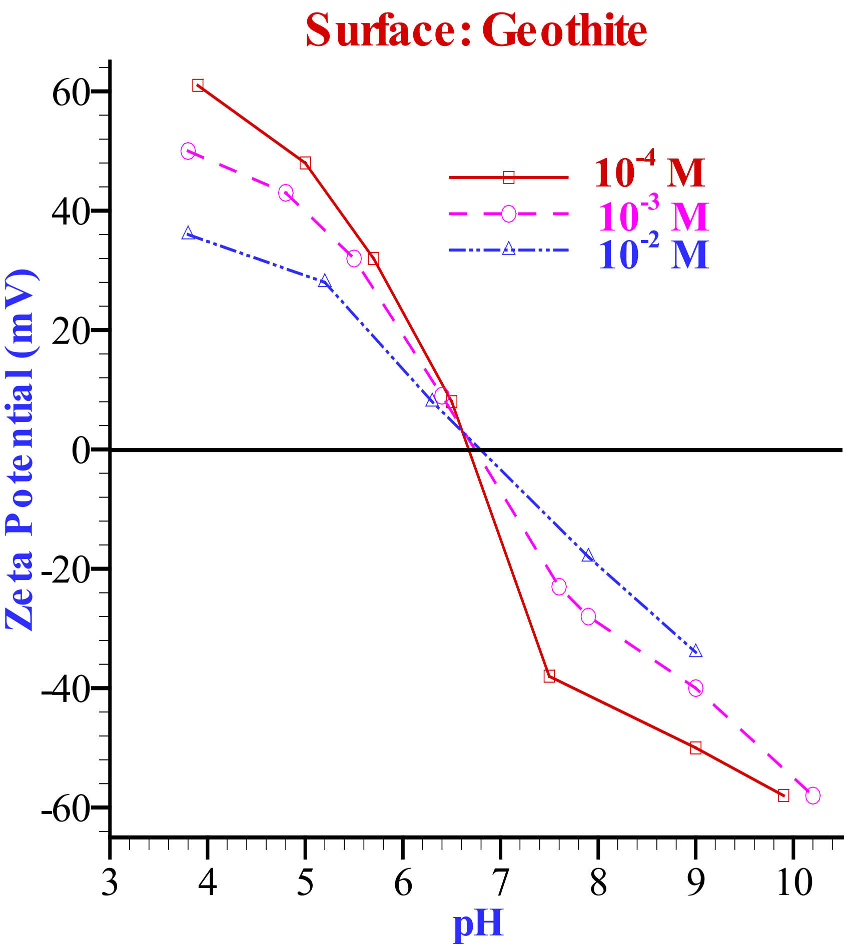
$$\nabla^2 \psi^* = \beta \sinh(\alpha \psi^*)$$

Nomenclature

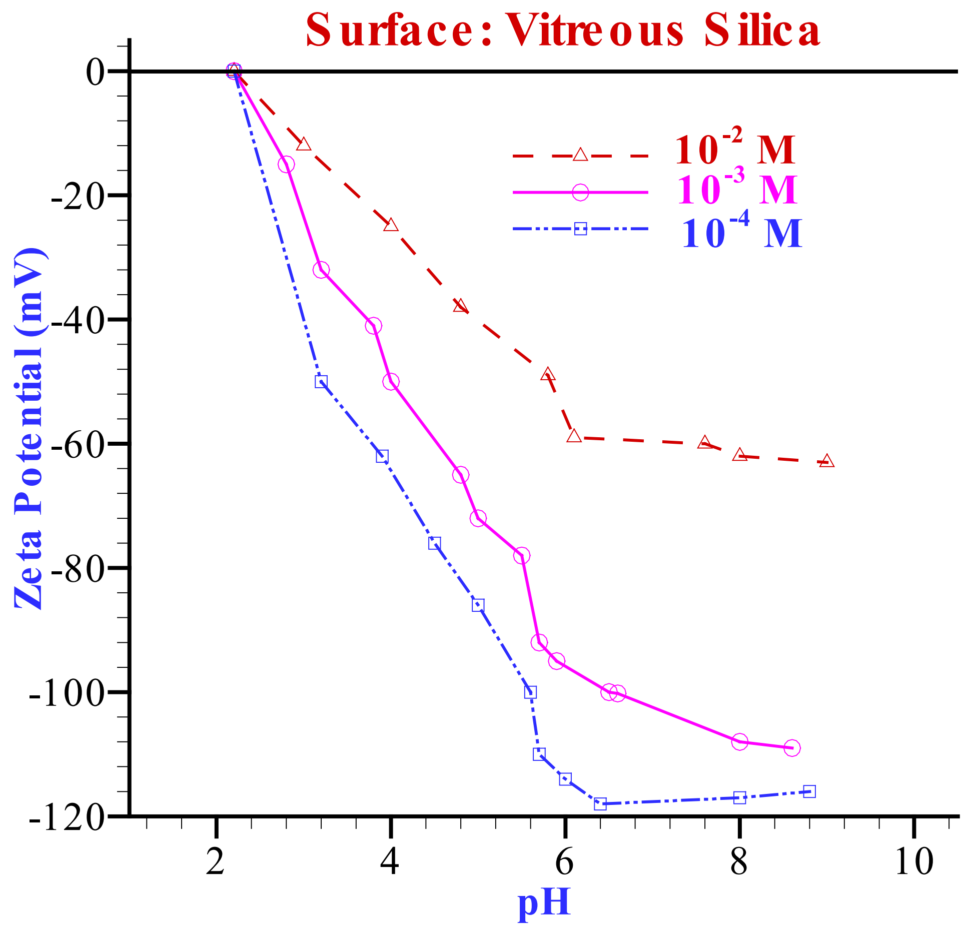
D	Dielectric constant
e	electron charge
k_b	Boltzmann constant
n_o	ion concentration
z	valence
α	ionic energy parameter
ε	$D/4\pi$
λ	Debye length
ρ_e	electric charge density
ϕ	electric field potential
ψ	electroosmotic potential
ω	Debye-Hückel parameter $1/\lambda$
ζ	zeta potential

Zeta Potential of Surface-Liquid Interface

$$\alpha = ez\zeta / k_b T \qquad \omega = \sqrt{\frac{8\pi n_o e^2 z^2}{D k_b T}} \qquad \beta = (\omega h)^2 / \alpha$$



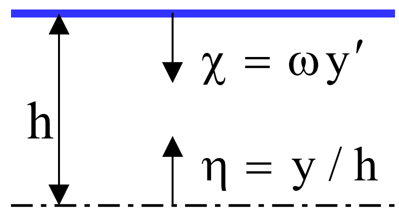
Source: Fuerstenau and Healy, 1972



Source: Wiese et al., 1971

Analysis of Poisson-Boltzmann Equation

(a) Poisson-Boltzmann Equation in 1-D



$$\frac{d^2\psi^*}{d\eta^2} = \beta \sinh(\alpha\psi^*)$$

(b) Derivative of the Electroosmotic Potential

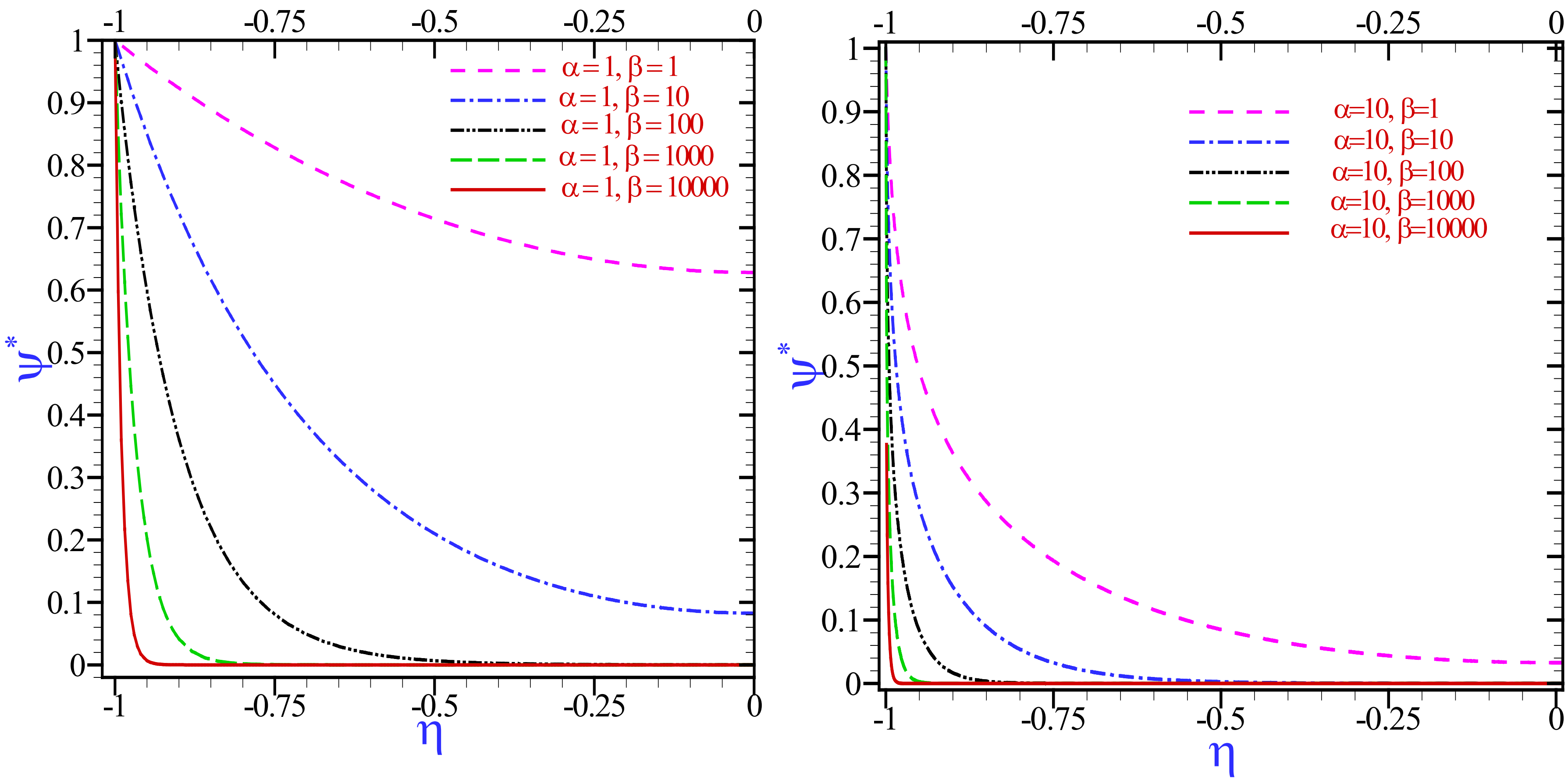
$$\frac{d\psi^*}{d\eta} = \sqrt{\frac{\beta}{\alpha}}\sqrt{2\cosh(\alpha\psi^*) - 2\cosh(\alpha\psi_c^*)}$$

(c) Potential Distribution across the Channel

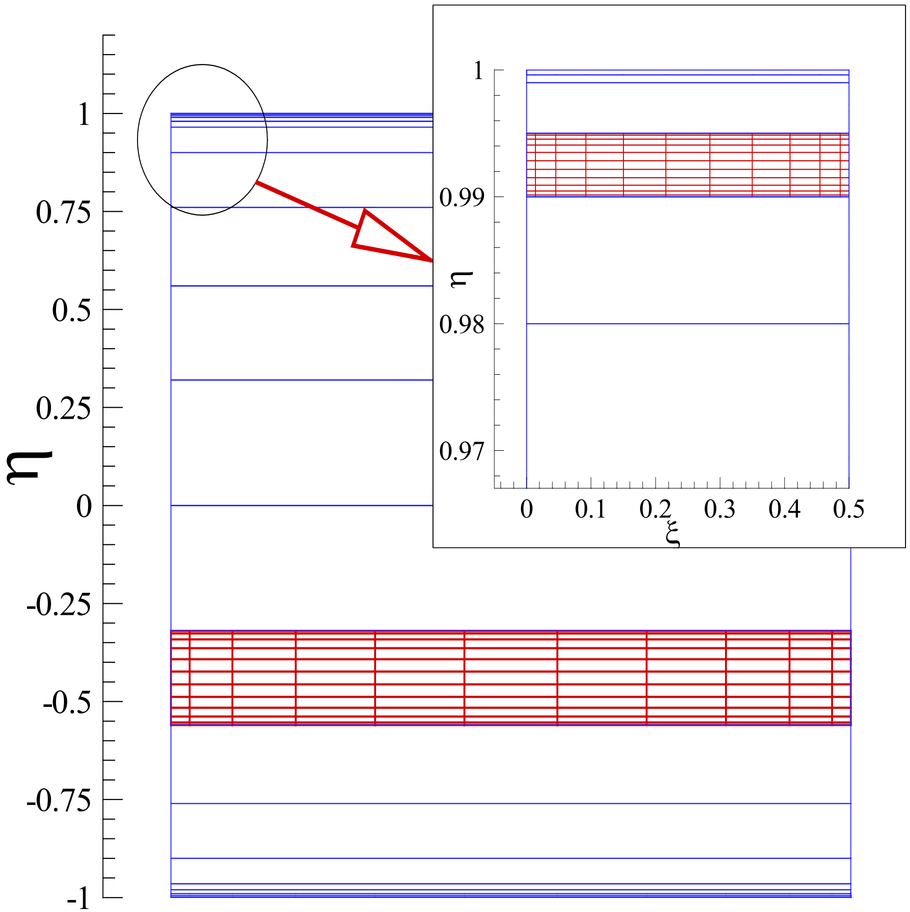
$$\psi^* = \frac{4}{\alpha}\tanh^{-1}\left[\tanh\left(\frac{\alpha}{4}\right)\exp\left(-\sqrt{\alpha\beta}\,\eta^*\right)\right]$$

$$\chi = \sqrt{\alpha\beta}\,\eta^* \qquad \chi = \omega y' = \omega h(1-\eta)$$

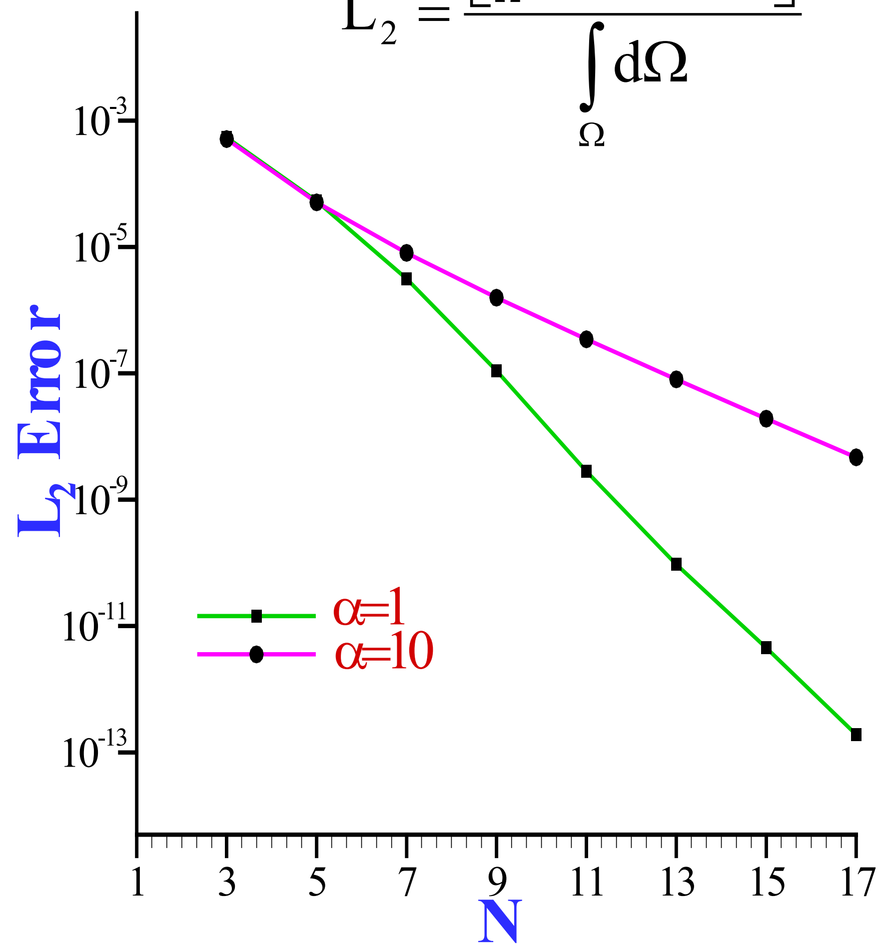
Potential Distribution Across the Channel



Spectral Convergence in Electric Double Layer



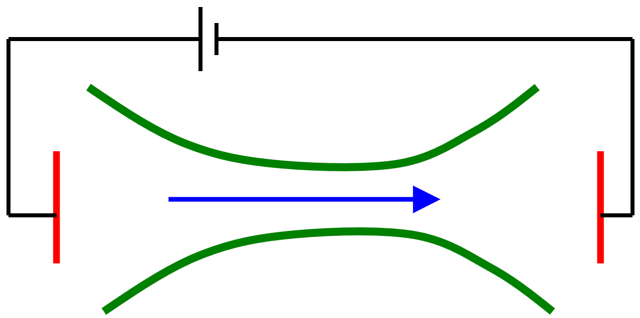
$$L_2 = \frac{\left[\int_\Omega R\left(\psi^*\right)^2 d\Omega\right]}{\int_\Omega d\Omega}$$



Governing Equations for Electroosmotic Flow

(a) Continuity Equation:

$$\nabla.\vec{u} = 0$$



(b) Navier Stokes Equations:

$$\rho_f\left(\frac{\partial\vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u}\right) = -\nabla P + \mu\nabla^2\vec{u} + \rho_e\vec{E}$$

Assumptions & Approximations

- Ion convection is negligible
- Ions are point charges
- Newtonian fluid
- Fluid properties are constant
- Medium is continuous
- Steady Flow

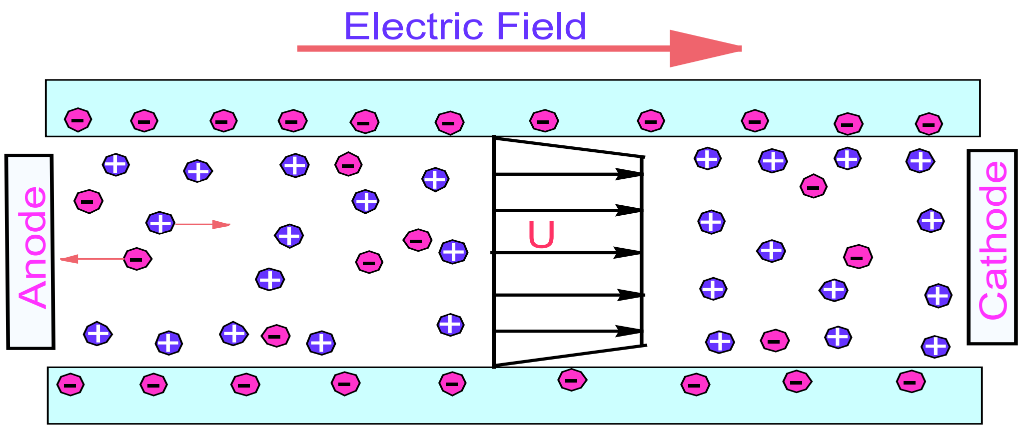
(c) Electric Field:

$$\nabla.(\sigma\nabla\phi) = 0$$
$$\vec{E} = -\nabla\phi$$

(d) Electroosmotic Field:

$$\nabla^2\psi^* = \beta\sinh(\alpha\psi^*)$$

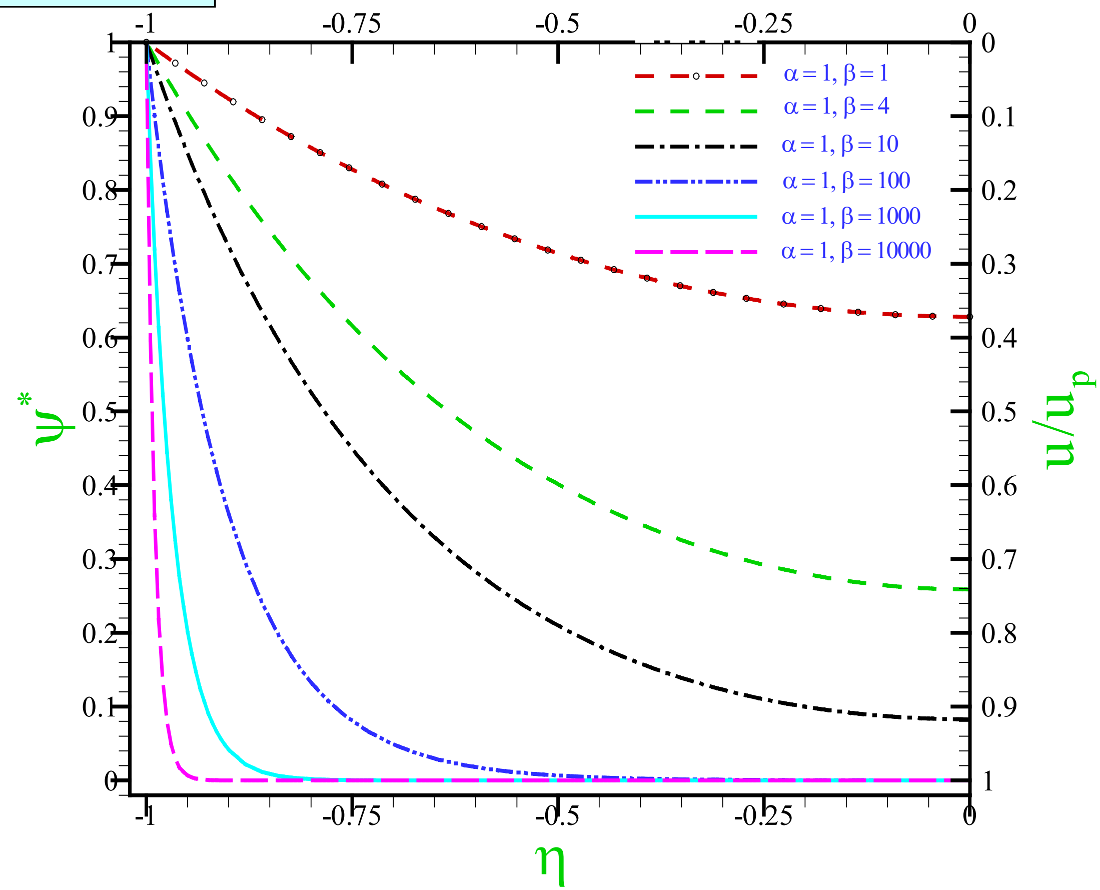
Pure Electroosmotic Flow in a Channel



$$\frac{\partial P^*}{\partial \xi} = \frac{d^2 U}{d\eta^2} + \frac{d^2 \psi^*}{d\eta^2}$$

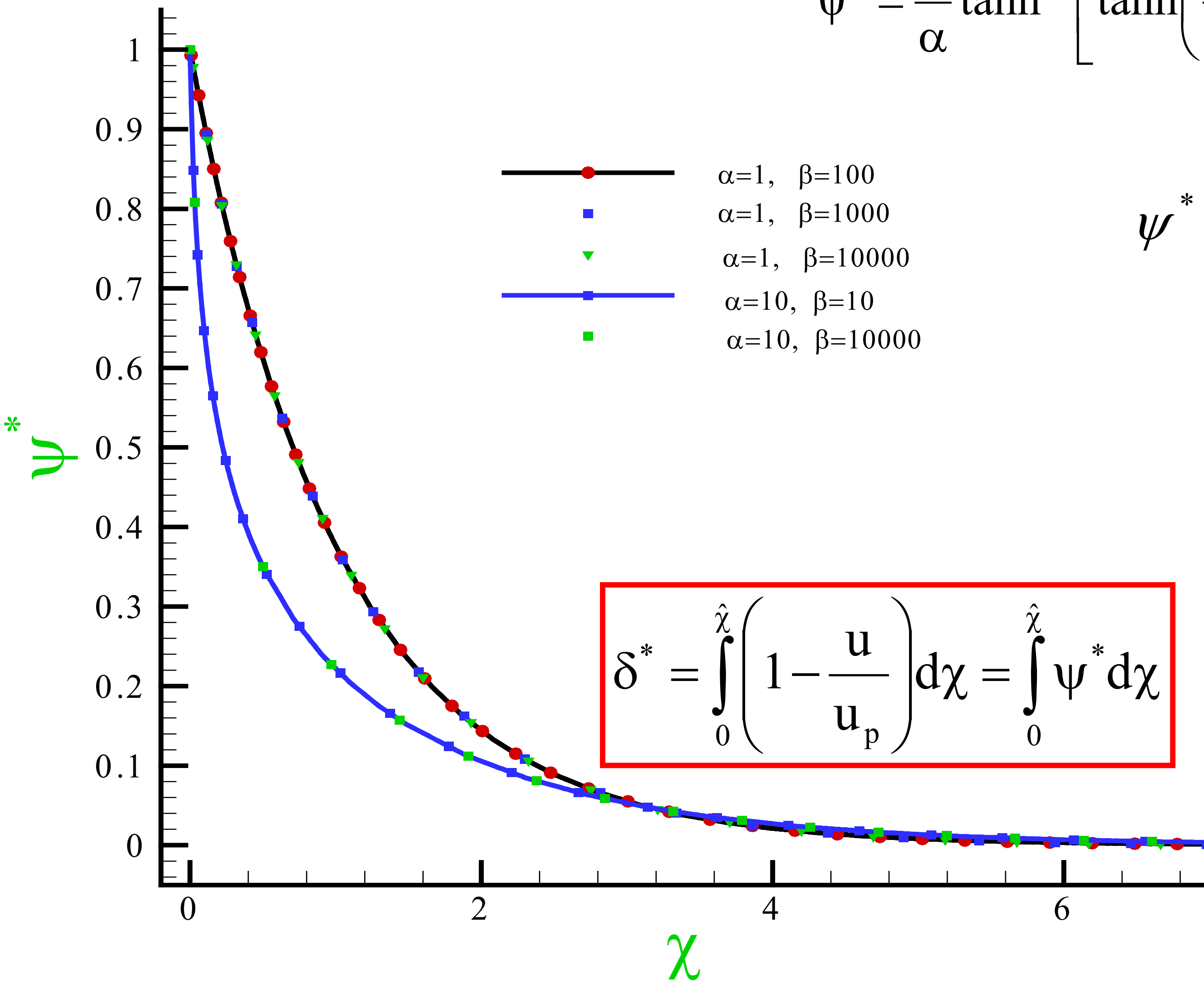
0

$$U = u / u_p = 1 - \psi^*$$



Potential Distribution Near the Wall

$$\psi^* = \frac{4}{\alpha}\tanh^{-1}\left[\tanh\left(\frac{\alpha}{4}\right)\exp(-\chi)\right]$$



$$\psi^* = \left(1 - \frac{u}{u_p}\right)$$

$$U = 1 - \psi^*$$

$$\chi = \omega y$$

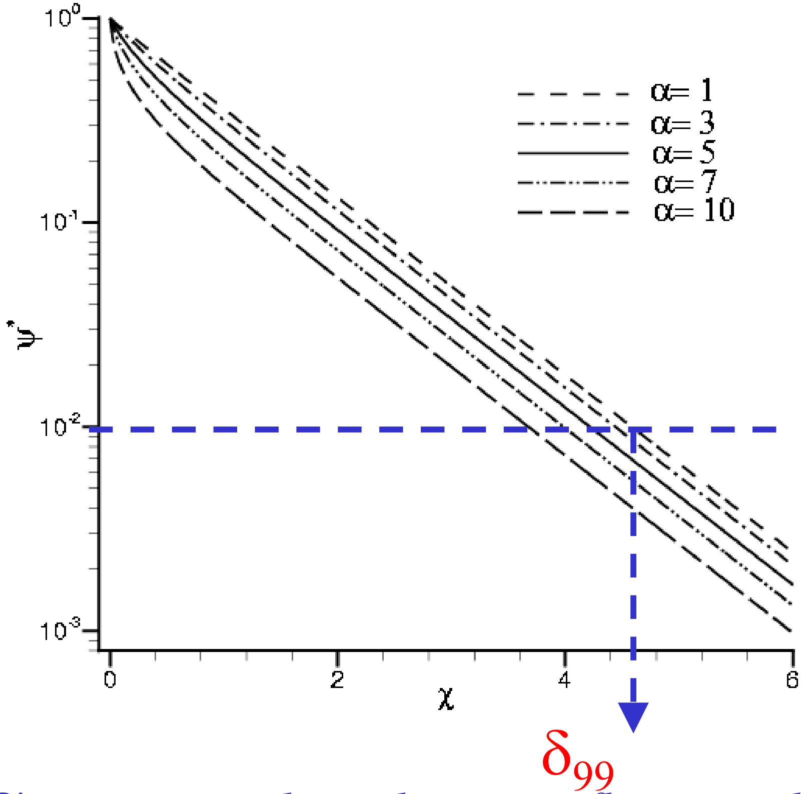
$$\delta^* = \int_0^{\hat{\chi}}\left(1 - \frac{u}{u_p}\right)d\chi = \int_0^{\hat{\chi}}\psi^* d\chi$$

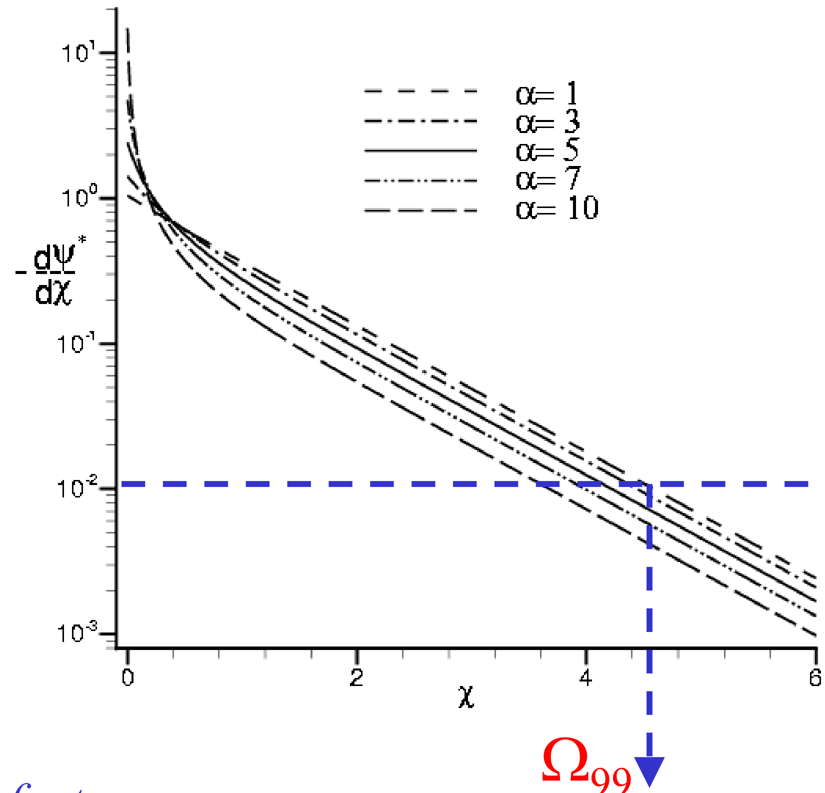
EDL Displacement Thickness

Dutta & Beskok *Analytical Chemistry*, 73(9); 1979-1986, 2001.

New Concepts in Electroosmotic Flows
Analogy to Boundary Layer Theory

Effective EDL Thickness δ_{99} — EDL Vorticity Thickness Ω_{99}



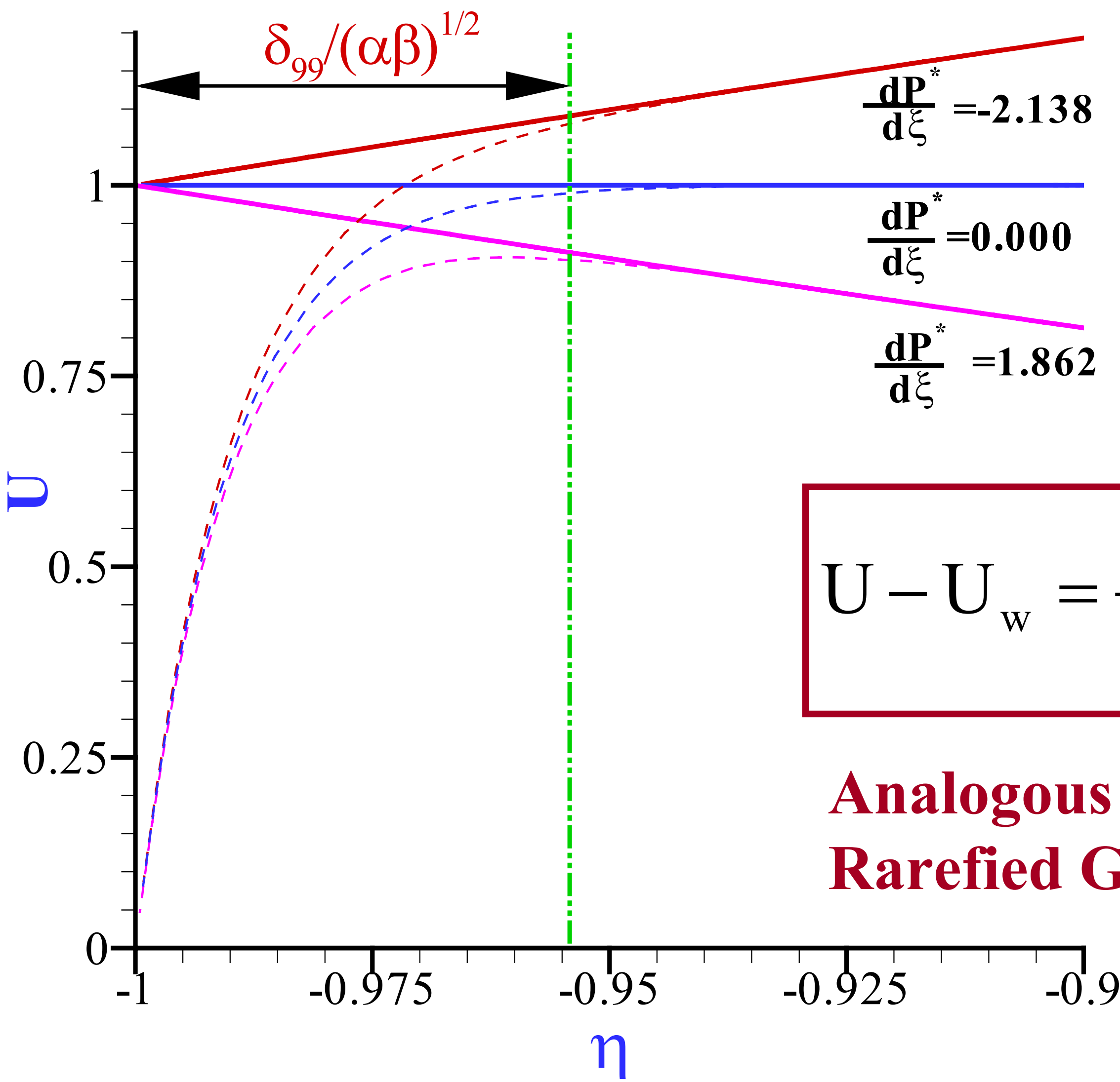


□ *δ^* expresses the volumetric flowrate defect*
Due to velocity distribution within the EDL

$$\chi = \omega y$$

Dutta & Beskok *Analytical Chemistry*, 73(9); 1979-1986, 2001.

EDL/Bulk Flow Matching Condition

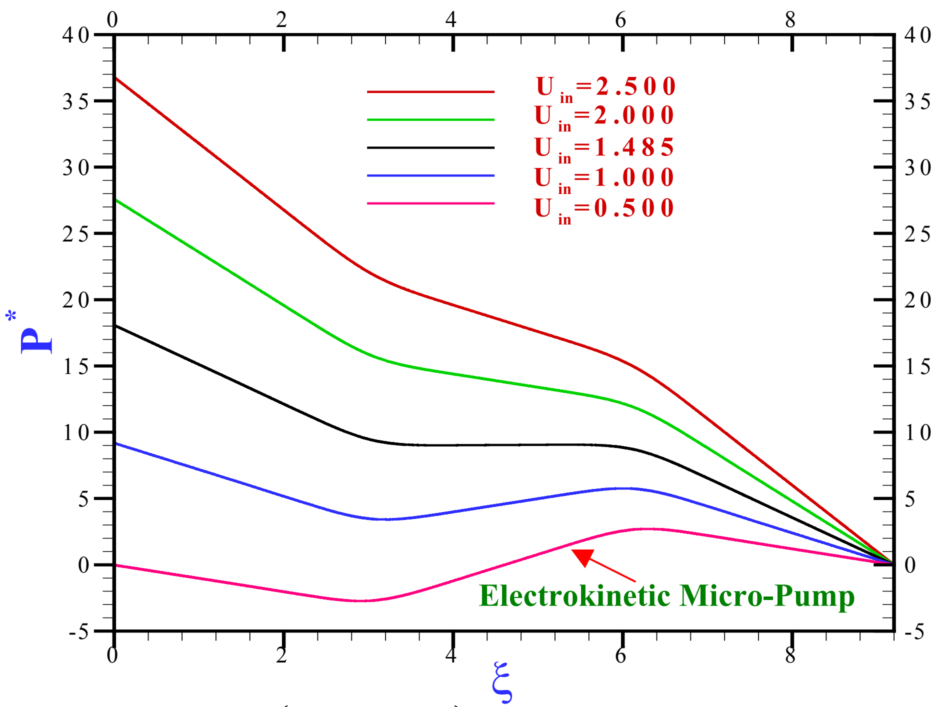


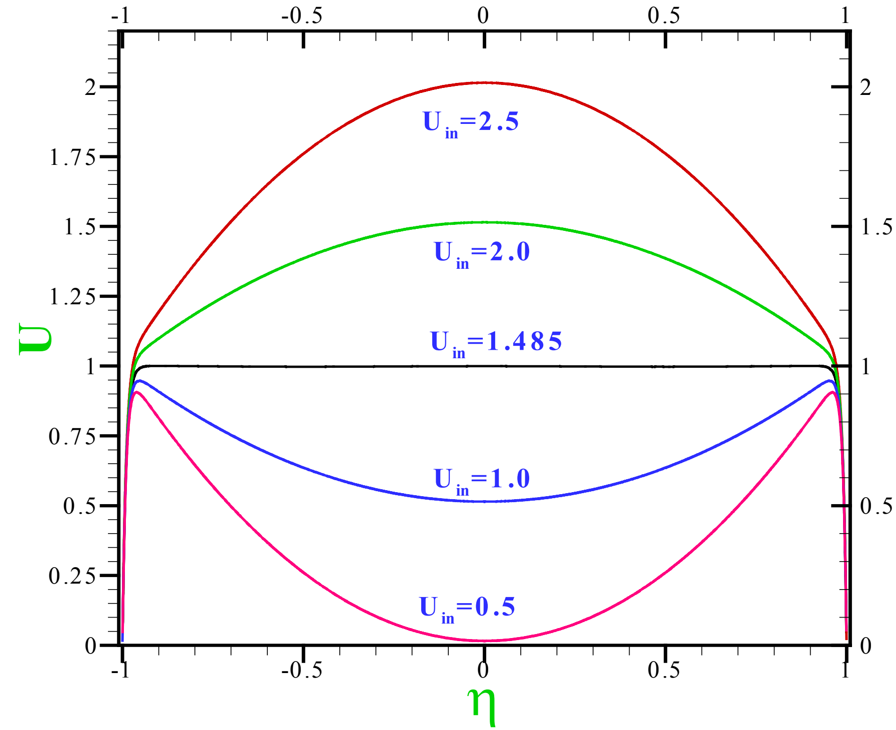
$$U_p = \frac{\zeta \varepsilon Y}{\mu}$$

$$U - U_w = \frac{\delta_{99}}{\sqrt{\alpha\beta}} \left.\frac{\partial U}{\partial n}\right|_w + U_p$$

Analogous to Slip Condition in Rarefied Gas Dynamics

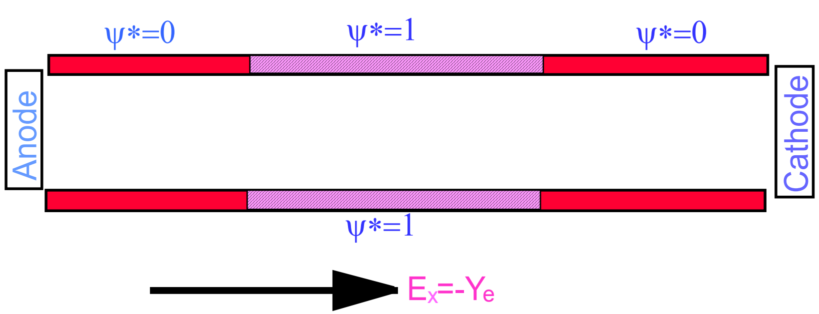
Mixed Electroosmotic/Pressure Driven Channel
Pressure & Velocity Distributions





$$\frac{dP^*}{d\xi} = 3\left(1-\frac{\delta^*}{\sqrt{\alpha\beta}}\right) - 2U_{in} \qquad U_{in} = \frac{u_{max}}{u_p}$$

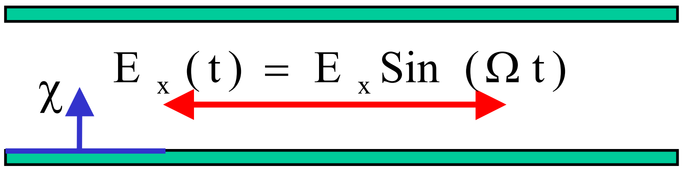
$$U(\eta) = -\frac{1}{2}\frac{dP^*}{d\xi}\left(1-\eta^2\right)+1-\psi^*(\eta)$$



Results are for α=1, β=10,000, Re=0.005

Time Periodic Electroosmotic Flows

$$\rho_f \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2} + \rho_e E_x \mathrm{Sin}(\Omega t)$$



$$\rho_e = -2 n_o e z \sinh(e z \psi / k_b T)$$

$$U = \frac{u}{u_p} \quad u_p = \frac{\zeta \varepsilon Y}{\mu} \quad \theta = \Omega t \quad \chi = \frac{y}{\lambda} = \omega y$$

$$\frac{\partial U}{\partial \theta} = \frac{1}{\kappa^2}\left[\frac{\partial^2 U}{\partial \chi^2} + \frac{\mathrm{Sin}(\theta)}{\alpha}\mathrm{Sinh}(\alpha \psi^*)\right]$$

$$\kappa = \lambda \sqrt{\frac{\Omega}{\nu}}$$

Dutta, Beskok *Analytical Chemistry,* **73(21);** 5097-5102, 2001.

Analytical Solution of Time Periodic EO Flow

$$U(\chi,\theta) = \mathrm{Im}[F(\chi).\exp(i\theta)]$$

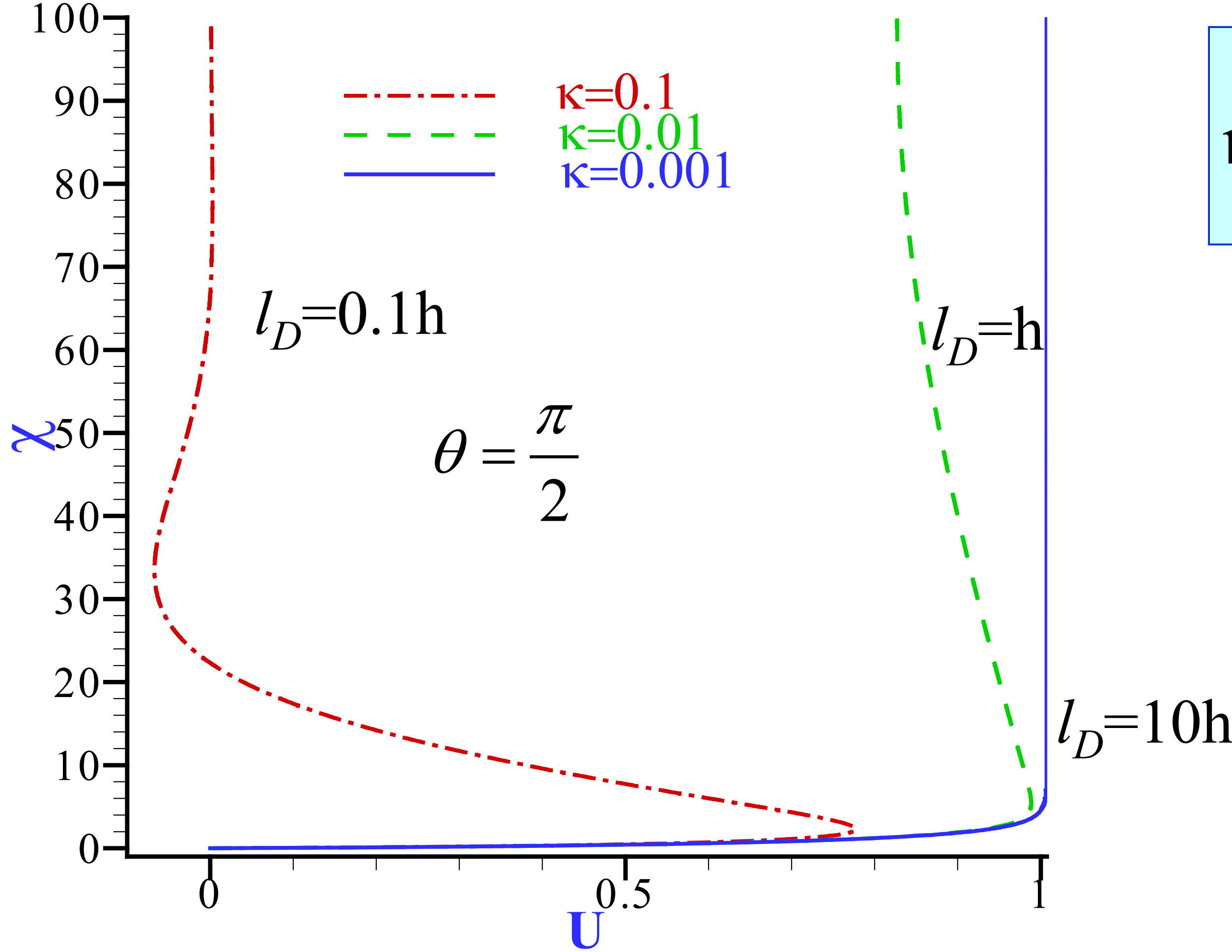
$$F(\chi) = 2A\mathrm{Sinh}(\sqrt{i\kappa}\chi) + [C(\chi) - D(\chi)]$$

$$C(\chi) = \frac{\exp(-\sqrt{i\kappa}\chi)}{2\sqrt{i\kappa}\alpha}\int_0^{\chi}\exp(\sqrt{i\kappa}\chi)\mathrm{Sinh}(\alpha\psi^*)d\chi$$

$$D(\chi) = \frac{\exp(\sqrt{i\kappa}\chi)}{2\sqrt{i\kappa}\alpha}\int_0^{\chi}\exp(-\sqrt{i\kappa}\chi)\mathrm{Sinh}(\alpha\psi^*)d\chi$$

$$A = \frac{\exp(-\sqrt{i\kappa}\chi)\int_0^{\infty}\exp(\sqrt{i\kappa}\chi)\mathrm{Sinh}(\alpha\psi^*)d\chi}{4\sqrt{i\kappa}\alpha\mathrm{Cosh}(\sqrt{i\kappa}\chi)} + \frac{\exp(\sqrt{i\kappa}\chi)\int_0^{\infty}\exp(-\sqrt{i\kappa}\chi)\mathrm{Sinh}(\alpha\psi^*)d\chi}{4\sqrt{i\kappa}\alpha\mathrm{Cosh}(\sqrt{i\kappa}\chi)}$$

Velocity Profiles



$$\kappa = \lambda\sqrt{\frac{\Omega}{\nu}}$$

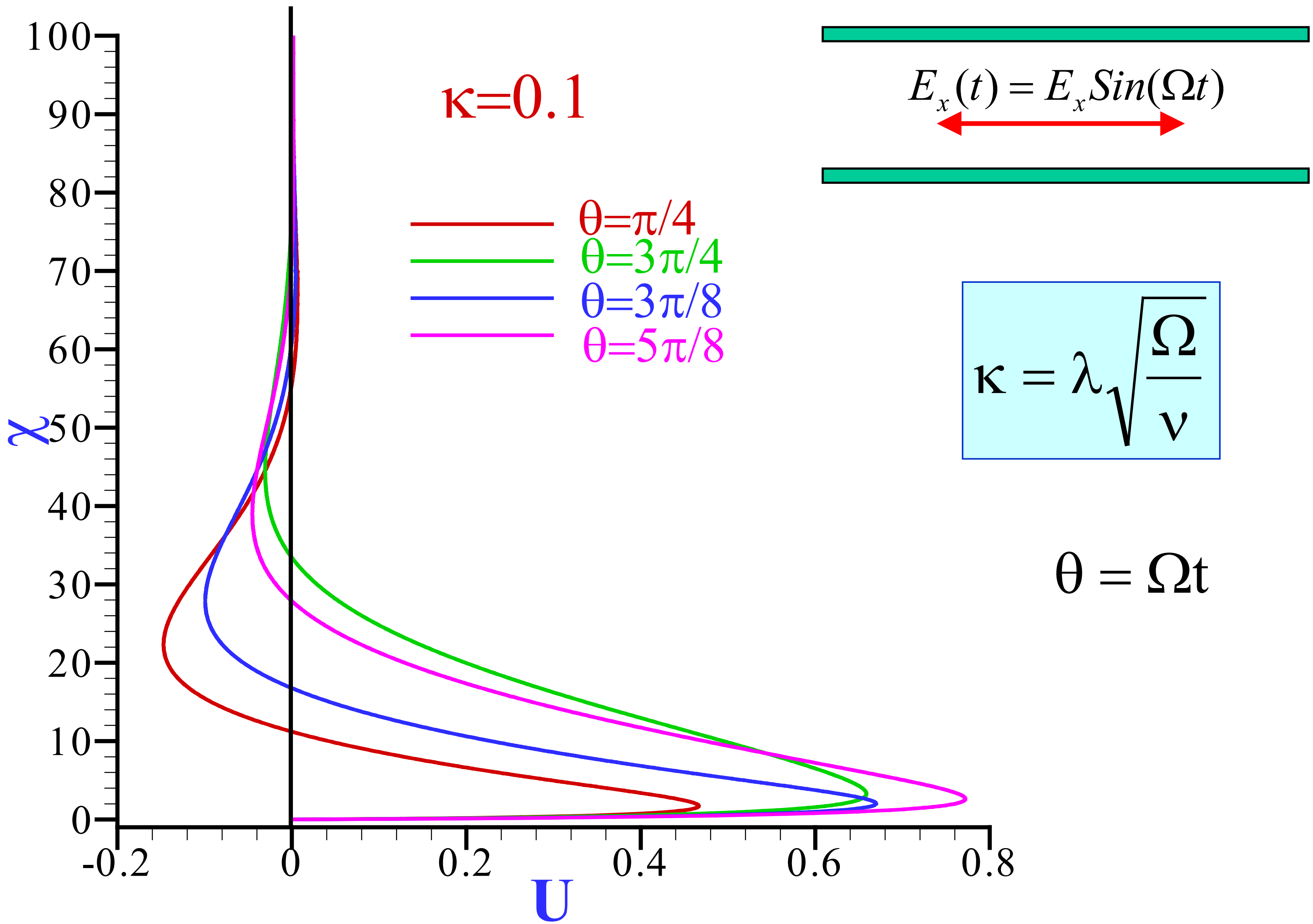
$$h = 100\lambda$$

$$\mathrm{U} = \frac{\mathrm{u}}{\mathrm{u}_{\mathrm{p}}}$$

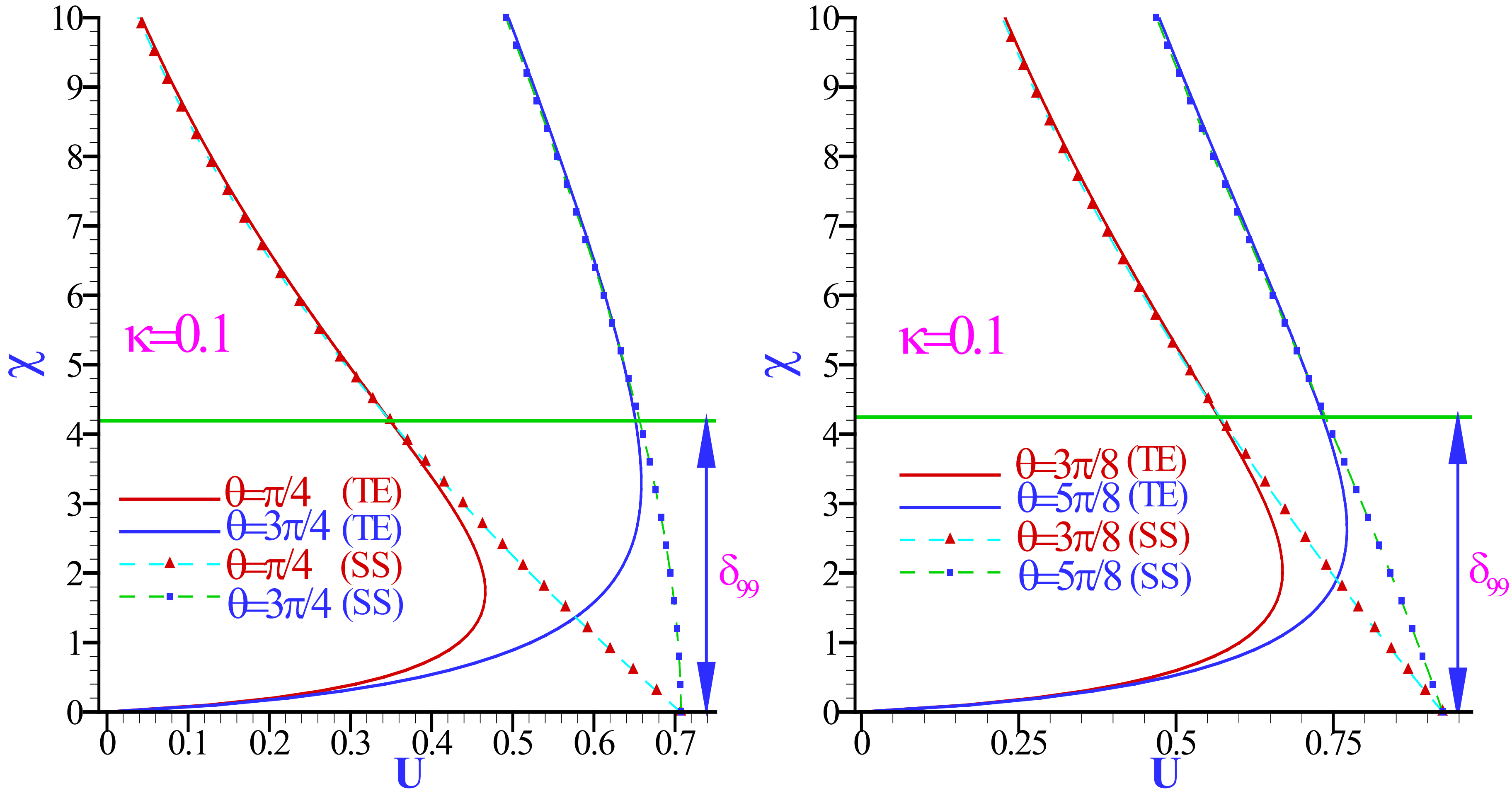
$$\mathrm{u}_{\mathrm{p}} = \frac{-\zeta\varepsilon\vec{\mathrm{E}}}{\mu}$$

- Non-irrotational flows for $l_D < \mathrm{h}$

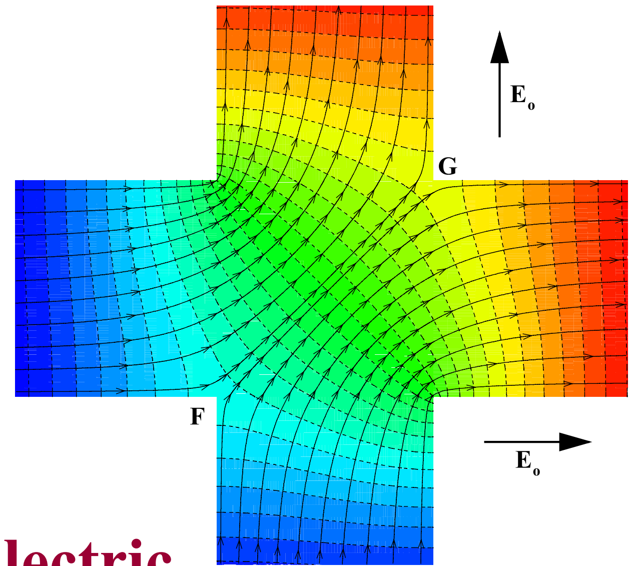
Velocity Distribution as a Function of Time



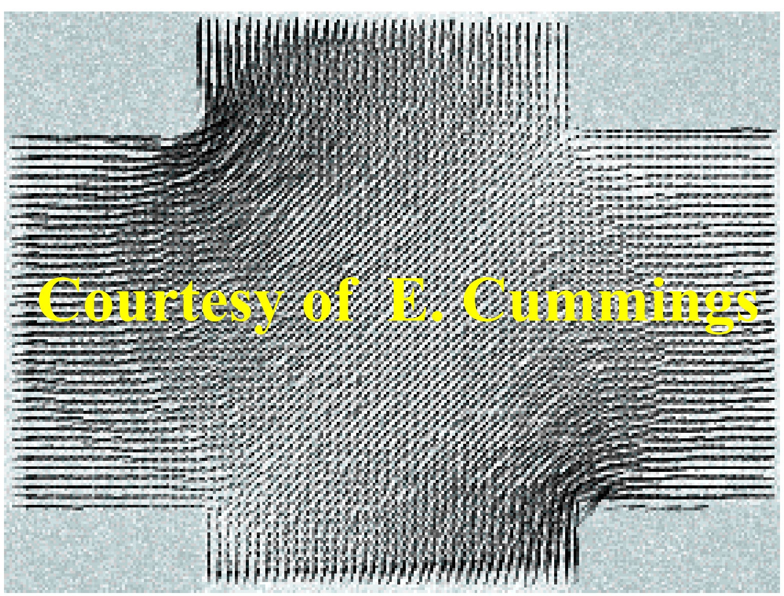
Near Wall Velocity Profiles: Comparisons to Bounded Stokes Layers

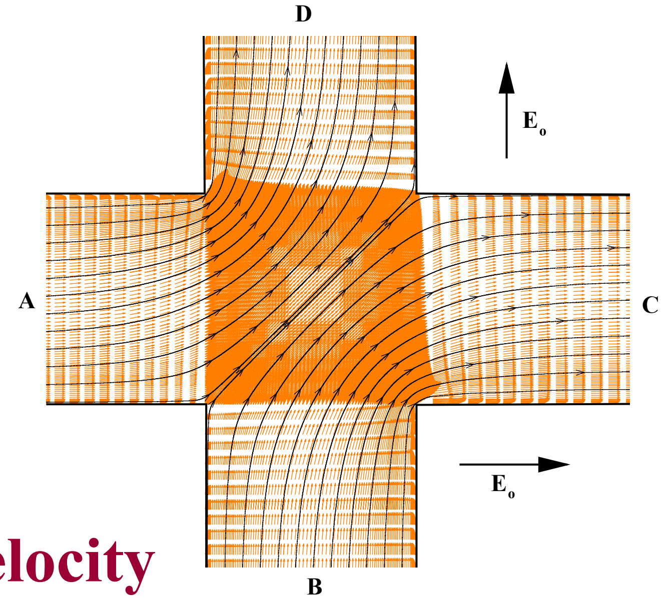


Electroosmotic Flow in a Cross-Channel

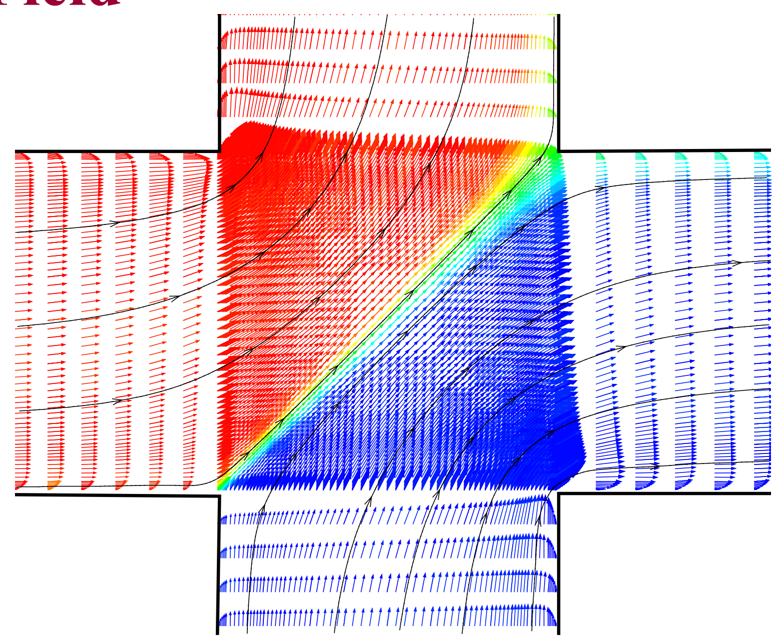


Electric Field

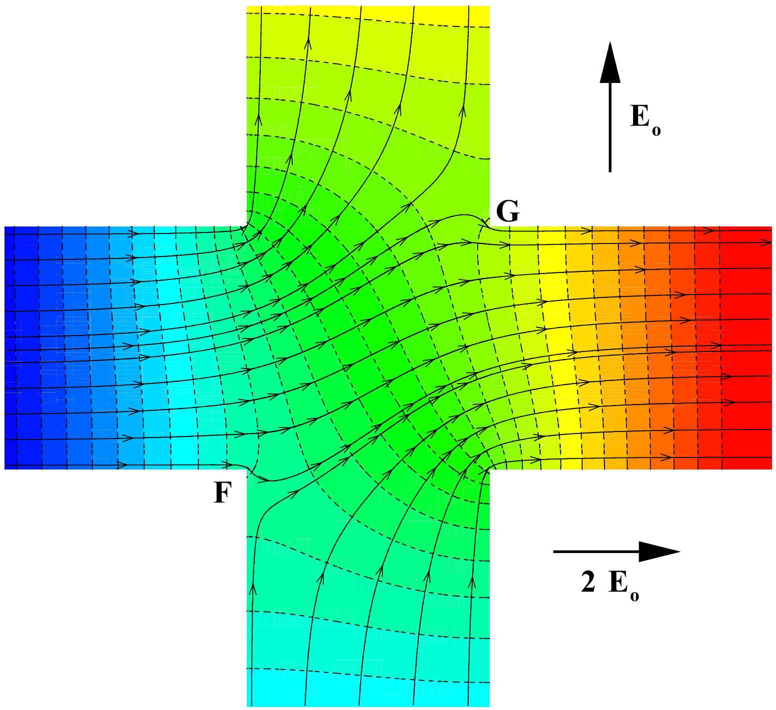


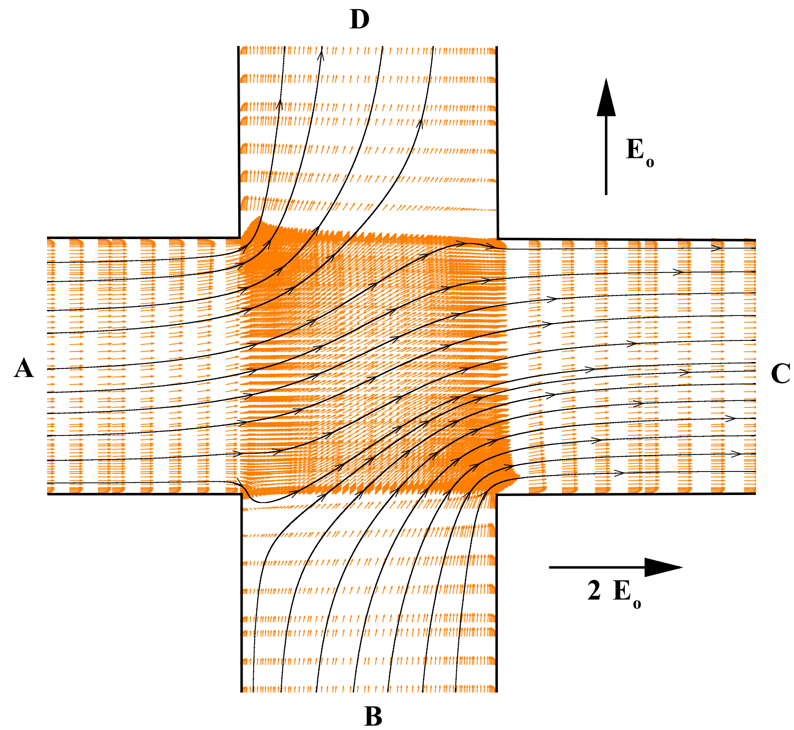


Velocity Field



Electroosmotic Flow *Control* in a Cross-Channel





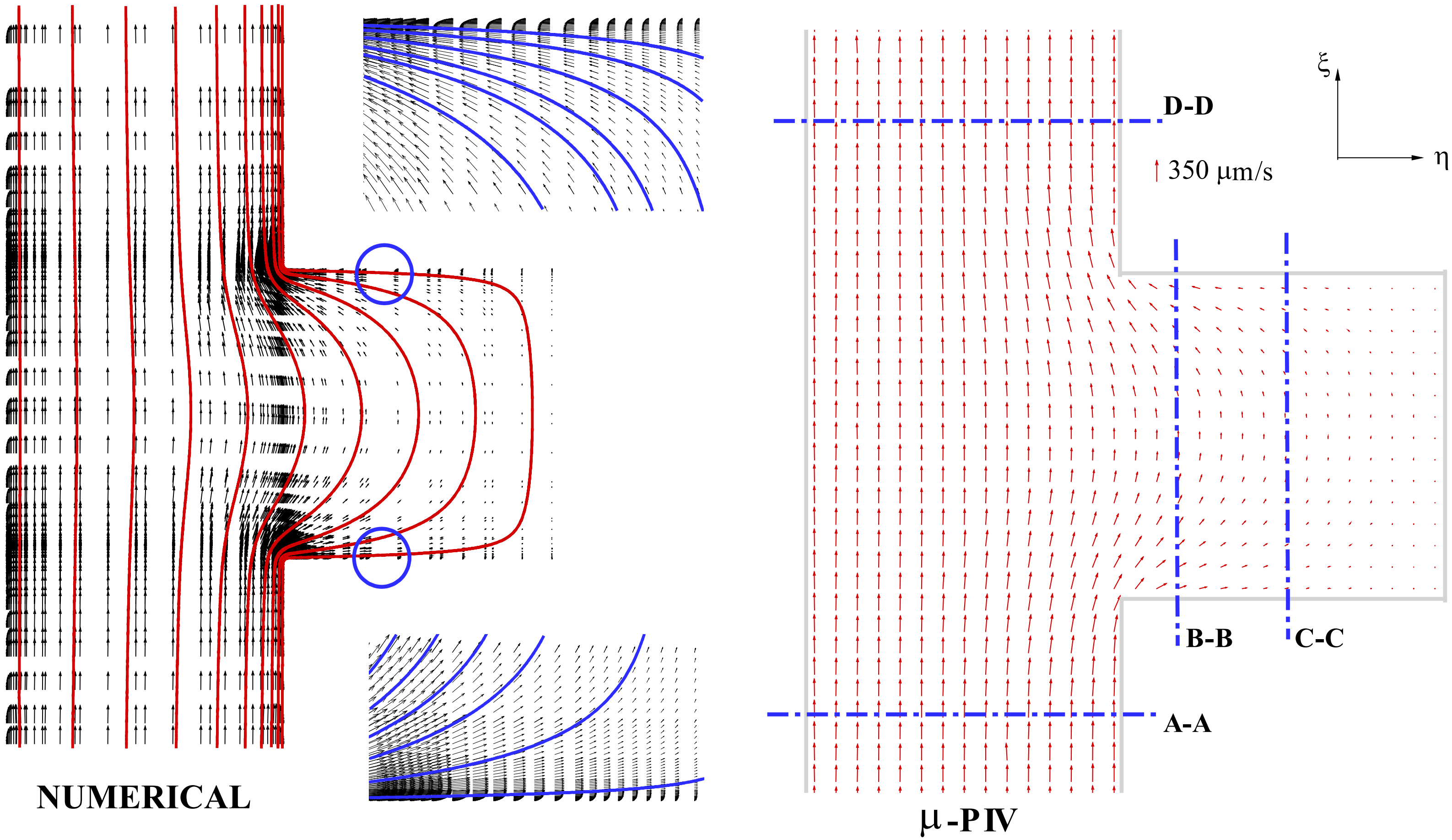
Electric Field

Velocity Field

Dutta, Beskok, Warburton *Journal of MEMS*, **11**(1), 36-44, 2002.

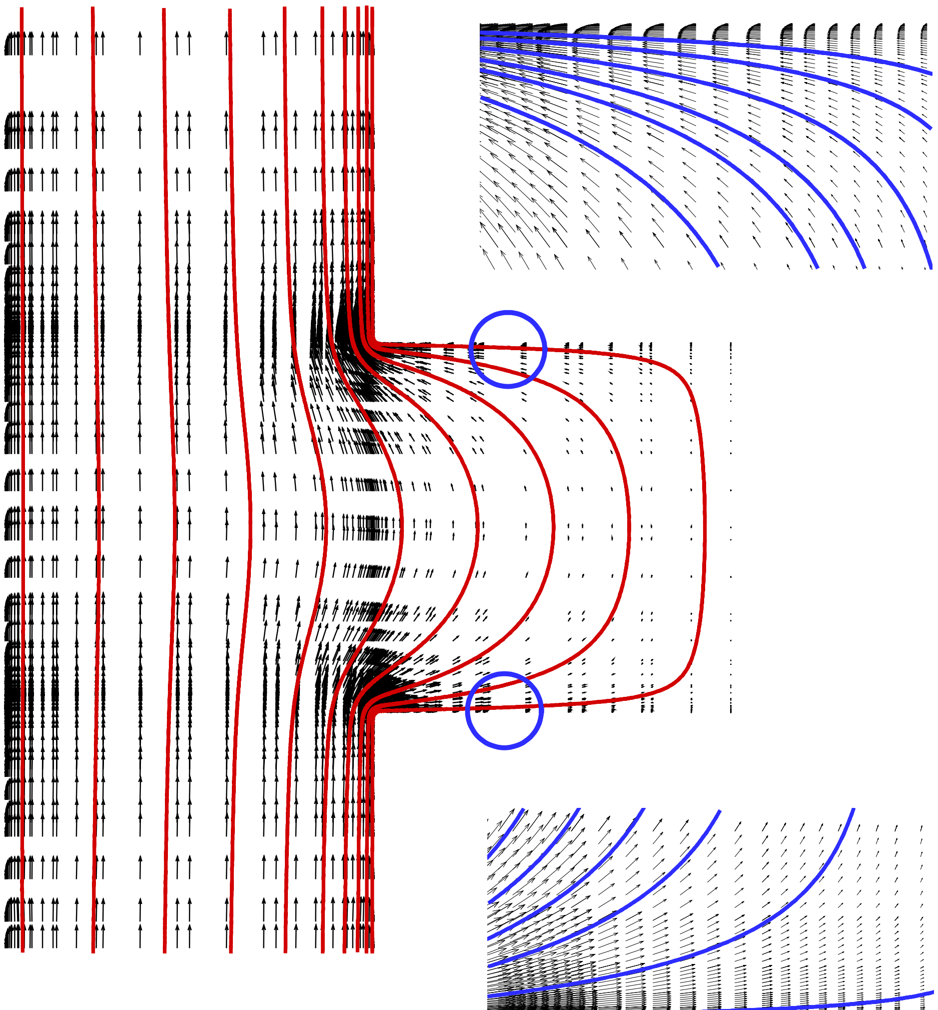
Comparison of Numerical Results with μ-PIV

Grooved Channel Results

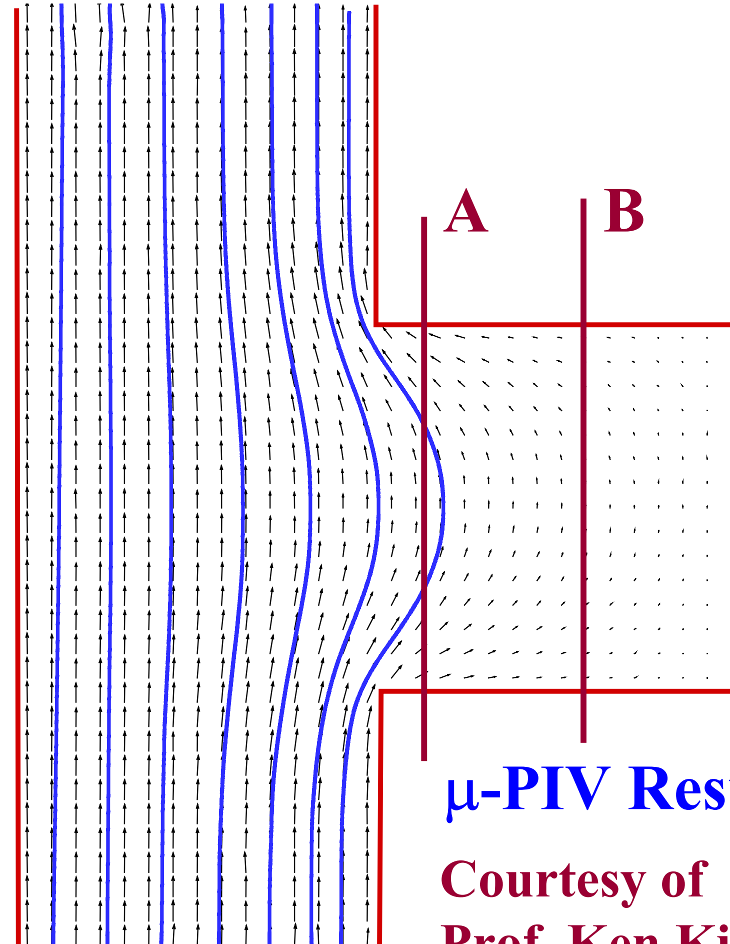


Kim, Kihm, Beskok, In Press, *Experiments in Fluids,* 2002

Electroosmotic Flow in a Grooved Channel



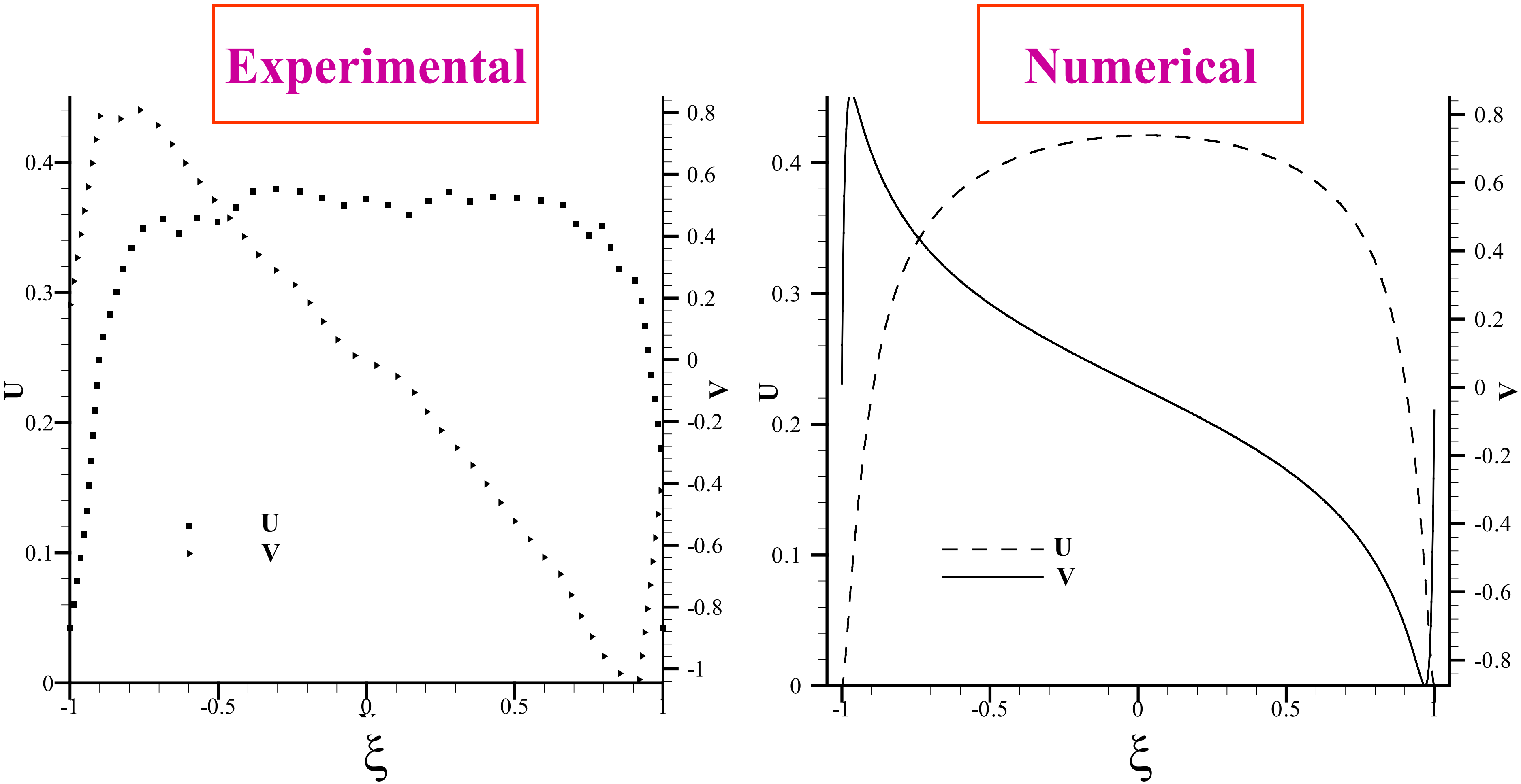
Numerical Results



μ-PIV Results

Courtesy of Prof. Ken Kihm TAMU

Point-wise Comparisons at Section A

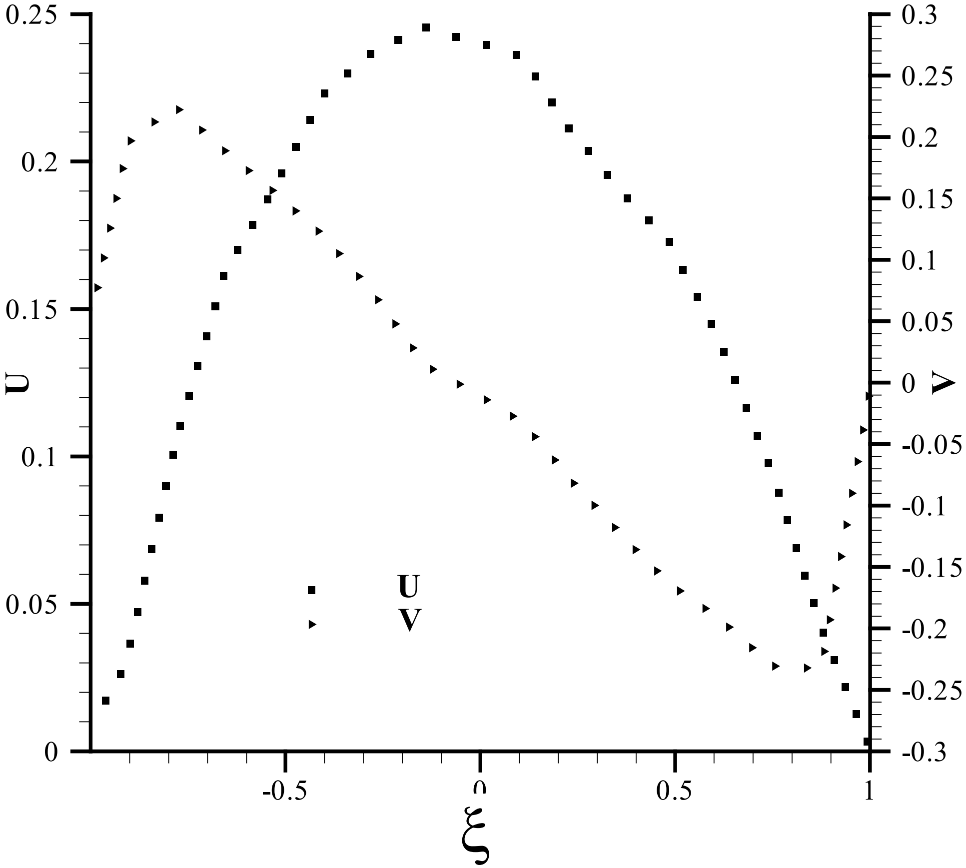


μ-PIV Results Courtesy of Prof. Ken Kihm, TAMU

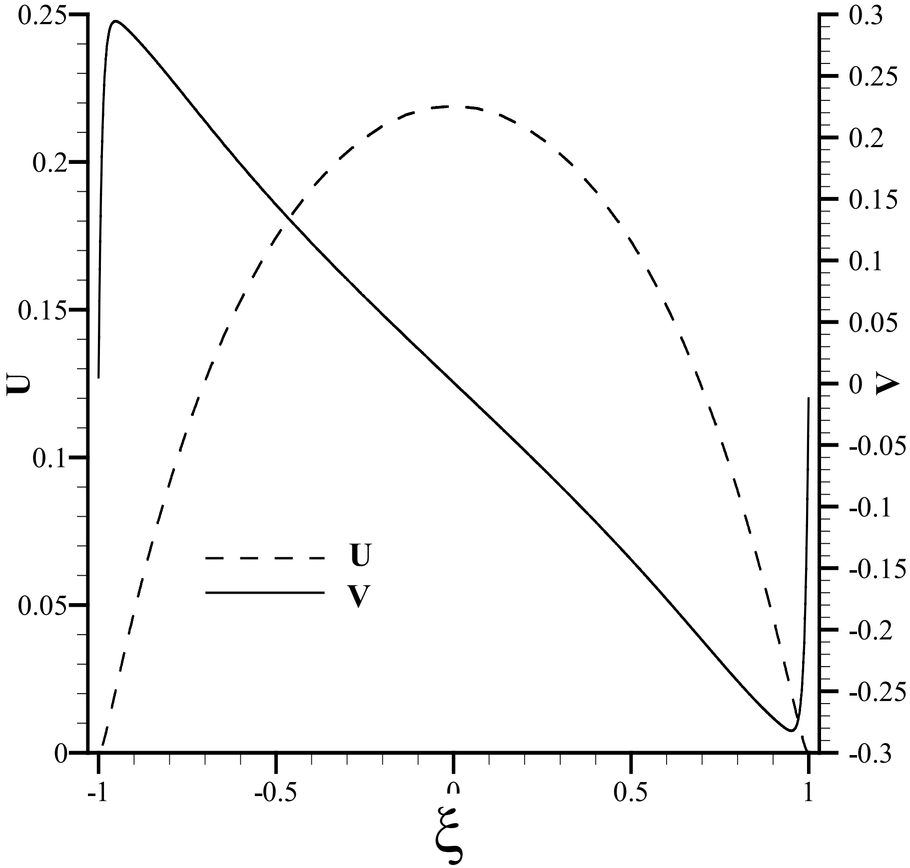
U: Stream Wise Velocity
V: Cross Stream Velocity

Point-wise Comparisons at Section B

Experimental



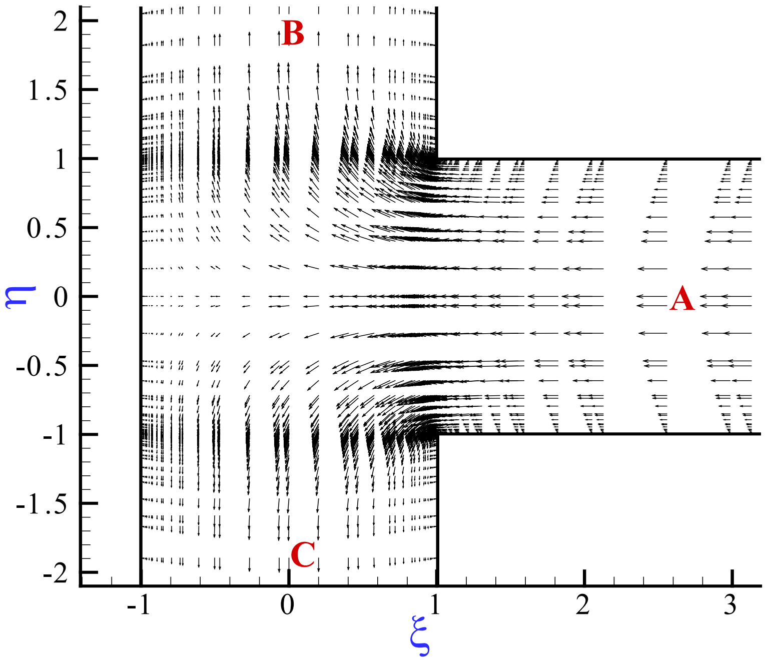
Numerical

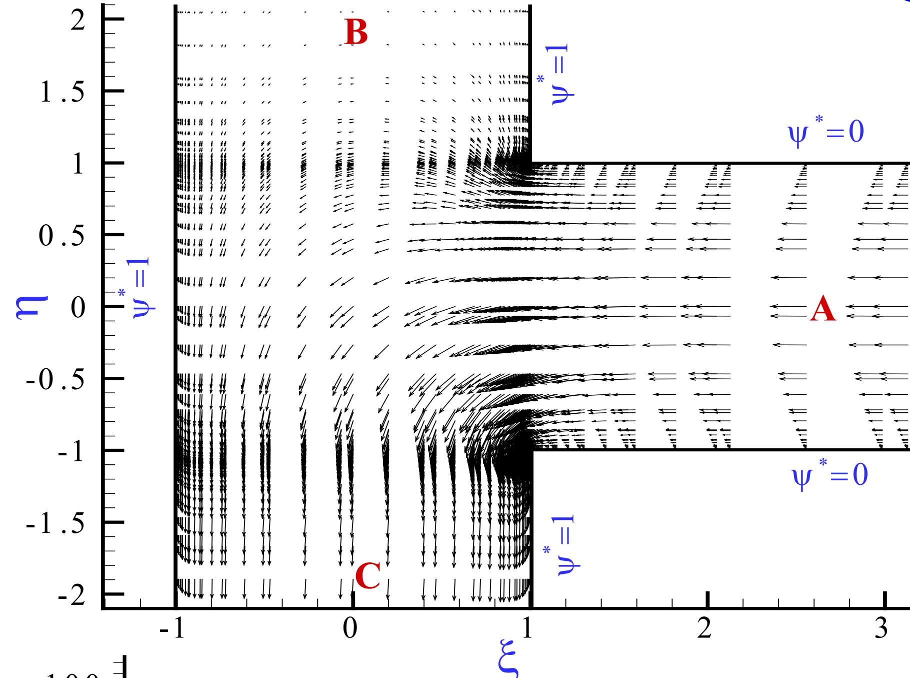


μ-PIV Results **Courtesy of**
Prof. Ken Kihm, TAMU

U: Stream Wise Velocity
V: Cross Stream Velocity

Flow Control in T-junction with Electroosmotic Forcing

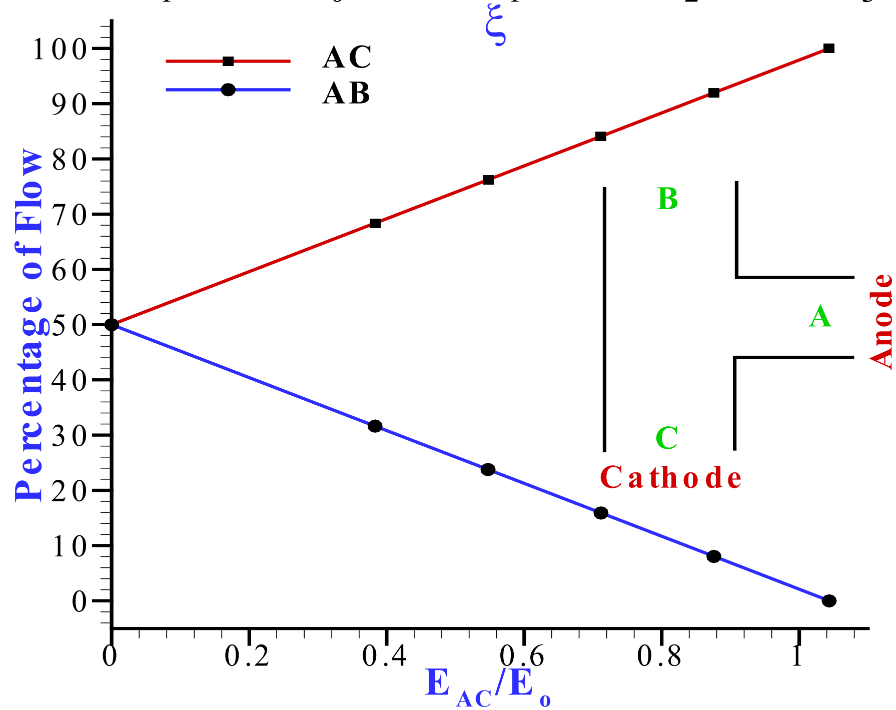




α=1 and β=10000

E_{AB}=0 and Re=0.005

Mixed Flow Case



Summary & Conclusions

- **Demonstrated *liquid pumping* with electroosmotic forces.**
- **Derived *analytical solutions* for velocity, pressure distribution and wall shear stress for mixed electrokinetic/pressure driven channels.**
- **Introduced *EDL displacement thickness*, *effective EDL thickness*, and *EDL/Bulk-Flow matching conditions*.**
- **Verified that Helmholtz-Smoluchowski velocity is the appropriate *slip boundary condition* for mixed electroosmotic/pressure driven as well as time-periodic electroosmotic flows.**
- **Demonstrated the possibility of *flow, scalar transport and separation control* in complex geometries.**
- **H/P element method enables resolution of very thin EDL with *exponential accuracy.***