1 Treating a Perturbing Potential

For a perturbing potential $A$, the free energy is given by

$$
e^{-\beta F(\lambda)} = \int dRe^{-\beta V-\beta \lambda A}$$

$$F(\lambda) = F(0) + \lambda \langle A \rangle_0 - \frac{\beta \lambda^2}{2} [\langle A^2 \rangle_0 - \langle A \rangle_0^2] + O(\lambda^3)$$

$$F(\lambda) = F(0) + \int_0^\lambda d\lambda' \langle A \rangle^t_{\lambda}$$

If $B$ is a property of the system,

$$B(\lambda) = B(0) - \beta \lambda [\langle AB \rangle_0 - \langle A \rangle_0 \langle B \rangle_0] + O(\lambda^2)$$

For example, let $A = \rho_k$ and $B = \rho_{-k}$ (density-density response). Then

$$\left. \frac{d\rho_{-k}}{d\lambda} \right|_0 = -\beta \langle |\rho_k|^2 \rangle = -\beta N S_k$$

2 Diffusion and velocity-velocity correlation.

The diffusion equation is

$$\frac{\partial \rho}{\partial t} = D \nabla^2 \rho(r, t)$$

$D$ can be determined from the mean-square displacement, or, equivalently, the velocity-velocity correlation time.

$$D = \lim_{t \to \infty} \frac{1}{6t} \langle (r(t) - r(0))^2 \rangle$$

$$= \frac{1}{3} \int_0^\infty dt \mathbf{v}(t) \cdot \mathbf{v}(0)$$

3 Treating a Dynamic Perturbing Potential

For an external field $A e^{-i\omega t}$, denote the response of $B$ as $\chi_{BA}(\omega)e^{-i\omega t}$. The fluctuation-dissipation theorem says

$$\chi_{BA}(\omega) = \beta \int_0^\infty dt e^{i\omega t} \langle B(t) \frac{dA(0)}{dt} \rangle$$

The energy absorption of the system is

$$\frac{dE}{dt} = \beta \left| \frac{\omega}{2} \right|^2 \int_0^\infty dt \cos(\omega t) \langle A(0)A(t) \rangle.$$ (10)

The dynamic structure factor (density-density response) is given by

$$S_k(\omega) = \frac{1}{2\pi} \int_{-\infty}^\infty dt F_k(t)e^{i\omega t}, \quad \text{where} \quad F_k(t) = \frac{1}{2} \langle \rho_k(t)\rho_{-k}(0) \rangle$$

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