Spatial Structure of Avalanches in Random Field Ising Model

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Supported by the National Science Foundation under Award Number DMR-03 25939 ITR,
via the Materials Computation Center at the University of Illinois at Urbana-Champaign
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Introduction

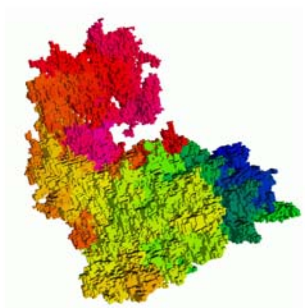


Fig.1. Fractal spatial structure of an avalanche. Fractal structures, as well as power laws, are characteristic of systems at their critical point. This moderate-sized avalanche involved the flipping of 282,785 spins in the simulation. The colors represent time: the first domains to flip are colored blue, and the last pink. Note that it has many branches and holes.

1. Avalanches are always associated with a slowly driven system exhibiting a sudden change of magnitude with extremely random properties: its appearance, size and duration are difficult to predict. In magnetic materials, these avalanches, corresponding to the reorganization of a region of spins, lead to the magnetic noise:the Barkhausen effect.
2. Previous work shows that the avalanche structure is visually interesting. First, it has fractal spatial structure (rugged on all scales). Second, it is anisotropic (it's longer than it is wider). Exciting experiments probing these spatial structures are coming on line.
3. Numerical studies of the fractal dimension and the anisotropy of avalanches are presented here. Corresponding quantities for clusters near critical field are also studied. Results are compared with that of the equilibrium studies.

Model

The zero-temperature non-equilibrium random-field Ising model (RFIM) has proven very successful in studying avalanches in disordered systems. The Hamiltonian is given by:

$$H = -J\sum_{\langle i,j\rangle} S_i S_j - \sum_i [H(t) + h_i] S_i$$

Here, the spins $S_i = \pm 1$ sit on a d-dimensional hypercubic lattice with periodic boundary conditions. The spins interact ferromagnetically with their z nearest neighbors with strength J, and experience a uniform external field H(t) and a random local field h_i. We choose units such that J=1. The random field h_i is distributed according to the Gaussian distribution:

$$\rho(h) = \frac{1}{\sqrt{2\pi}R} e^{-h^2/2R^2}$$

The external field H(t) is increased arbitrarily slowly from $-\infty$ to ∞. The dynamics of our model includes no thermal fluctuations: each spin flips deterministically when it can gain energy by doing so. That is, it flips when its local field changes sign. This change can occur in two ways: a spin can be triggered when one of its neighbors flips (by participating in an avalanche), or a spin can be triggered because of an increase in the external field H(t) (starting a new avalanche, domain nucleation).

Result 1: Fractal dimension: d_s

The avalanche is rugged on all scales, which naturally indicates that its surface will be fractal: $S \sim l^{d_s}$, where S is the area of the outermost surface of the avalanche, l is the length scale of the avalanche and d_s is the fractal dimension. We assume the volume enclosed by the outermost surface of the avalanche scales in a manner numerically consistent with this volume being nonfractal: $v \sim l^d$. Therefore, we have:

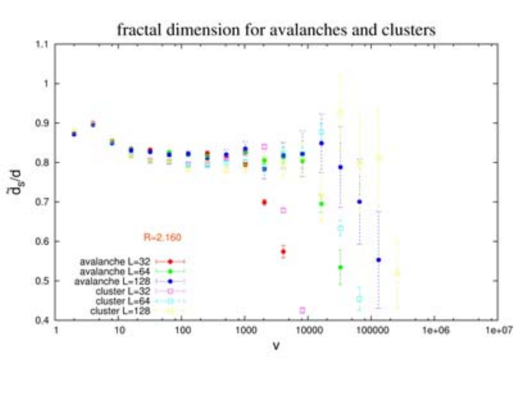
$$S \propto v^{d_s/d}$$

In this work, we focus on the critical point of the system where crackling noises with avalanches of all sizes appear. So we can calculate the fractal dimension with a wide range of avalanche volume. We bin the avalanches by the volume v, logarithmically spaced by powers of 2. Averaging over bins and samples gives the mean surface area. Then estimates of the dimension of these surfaces, i.e. d_s can be obtained by taking the discrete logarithmic derivative:

$$\tilde{d}_s(R,L,v) = \ln[\bar{S}(R,L,\sqrt{2}v)/\bar{S}(R,L,v/\sqrt{2})]/\ln 2$$

To show the difference of the spatial structure between avalanches and clusters in the non-equilibrium RFIM, we also calculate the fractal dimension of the clusters at states near the critical field H_c. See Fig.2.

Fig.2. Dependence of the outermost surface area of avalanche on the enclosed volume v is expressed as an effective exponent at R=2.160 ~ R_c, for L=32,64,128. The avalanche surface area scales as $v^{0.822 \pm 0.007}$ for the largest avalanches that are no affected by the finite-size effects, yielding a fractal dimension d_s=2.47 $\pm$ 0.02 for the avalanche surfaces. For clusters at H ~ H_c (L), we have d_s=2.38 $\pm$ 0.02.



Result 2: Anisotropy: A

The anisotropy of a given size avalanche or cluster can be determined from its radius of gyration tensor R_{ijk}^2 (i,j,k=x,y,z in 3D), by diagonalization of the tensor and calculation of the principal radii of gyration. The anisotropy (A) is then defined to be the ratio R_{min}^2/ R_{max}^2, where R_{min}^2 and R_{max}^2 are the smallest and the largest eigenvalues of the radius of gyration tensor, respectively. The numerical result is shown in Fig.3. First, we notice that for both the cluster and the avalanche, there is a regime of nearly two decades in s ($10 \sim 10^3$) over which finite-size effect is almost negligible. Below this regime, there are some abnormal points due to the discrete size effect. Above this regime, finite-size effect leads to big statistical error. Second, comparing with the avalanches, the clusters show unambiguously different behaviors for the intermediate size. Quantitatively, we can fit data in this regime to tell the difference. Considering the leading non analytic and analytic correction to scaling, we can write the asymptotic form of the principal radii of gyration as

$$\langle R_{s,\sigma}^2 \rangle = r_\sigma s^{2\nu}(1 + a_\sigma s^{-\theta} + b_\sigma s^{-1} + \cdots)$$

where σ =min or max. The coefficients r_σ, a_σ and b_σ are independent of s, ν is the leading scaling exponent and is equal to the inverse of the fractal dimension d_s, $s^{-\theta}$ is the leading nonanalytic correction-to-scaling term, and s^{-1} represents the leading analytic correction-to-scaling term. From this, we can write the anisotropy as

$$\langle A_s \rangle = \langle R_{s,\min}^2 / R_{s,\max}^2 \rangle = A_\infty(1 + as^{-\theta} + bs^{-1} + \cdots)$$

where $a=a_{min}-a_{max}$ and $b=b_{min}-b_{max}$. The fitting result is also shown in Fig.3. We notice that for the non-equilibrium case, the cluster has lower A_∞ than that of the avalanche. This means that clusters are more anisotropic than avalanches in non-equilibrium.

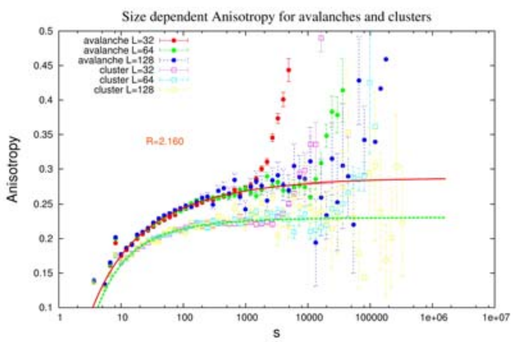


Fig.3. Dependence of the Anisotropy (A) of avalanches (clusters) on the sizes expressed as A(R,L,s), at R=2.160 ~ R_c, for L=32,64,128. The fitting result for the intermediate-size avalanches (clusters) with $10<s<10^3$ follows:

For avalanches:
$A_\infty = 0.29 \pm 0.01$; $a = -0.77 \pm 0.24$;
$\theta = 0.37 \pm 0.11$; $b = -0.59 \pm 0.44$.

For clusters:
$A_\infty = 0.23 \pm 0.01$; $a = -0.39 \pm 0.48$;
$\theta = 0.42 \pm 0.39$; $b = -1.48 \pm 0.81$.

Conclusions

The equilibrium study of the cluster surface has been done by Middleton and Fisher. Fractal dimension of the cluster surface is obtained for the ground state of RFIM. But the anisotropy study is still absent. As for the avalanche in the equilibrium model, so far, there has been no reports about the fractal dimension and the anisotropy. We summarize the known results in the following table:

	Equilibrium	Non-equilibrium
cluster:	$d_s^{c-eq} = 2.27 \pm 0.02$	$d_s^{c-neq} = 2.38 \pm 0.02$
	$A_\infty^{c-eq} = ?$	$A_\infty^{c-neq} = 0.23 \pm 0.01$
avalanche:	$d_s^{a-eq} = ?$	$d_s^{a-neq} = 2.47 \pm 0.02$
	$A_\infty^{a-eq} = ?$	$A_\infty^{a-neq} = 0.29 \pm 0.01$

We notice that for the cluster, it has lower fractal dimension in equilibrium than in non-equilibrium. And for the non-equilibrium case, the cluster has lower fractal dimension and higher anisotropy than that of the avalanche.

References

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