Population Dynamics in Disordered Media

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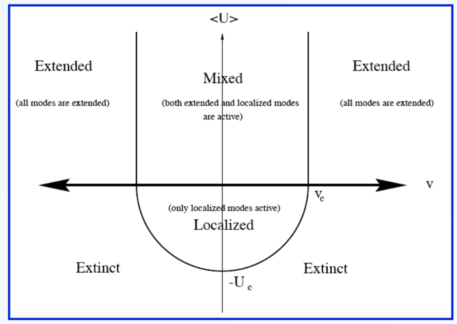
Population Biology

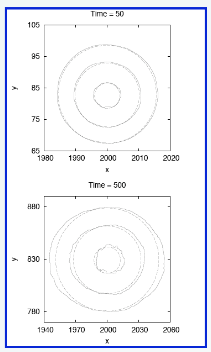
- The concentration of a bacterial species c(**x**, t) can be described by a reaction-diffusion equation:

$$\frac{\partial c(\mathbf{x},t)}{\partial t} = D\nabla^2 c(\mathbf{x},t) - \mathbf{v}\cdot\nabla c(\mathbf{x},t) + (a + U(\mathbf{x}))\,c(\mathbf{x},t) - bc^2(\mathbf{x},t)$$

- U(**x**) is a random variable corresponding to a "nutrient concentration," and introduces disorder into the system.
- It is useful to compare the problem to that of vortex lines in superconductors with columnar defects and hopping conduction in semiconductors.
- One dimensional problem of population growth in the presence of a single nutrient "oasis" has been well-studied, with the phase diagram shown—the system exhibits a delocalization transition.



Phase diagram for 1D problem with a single nutrient oasis. Taken from [1].



Disorder-averaged concentration contours for the 2D spreading problem. For small times the average behavior is diffusive, but at longer times it becomes superdiffusive. The diffusive exponent α is measured to be .585±.002. Taken from [4].

High Convection Velocity Limit

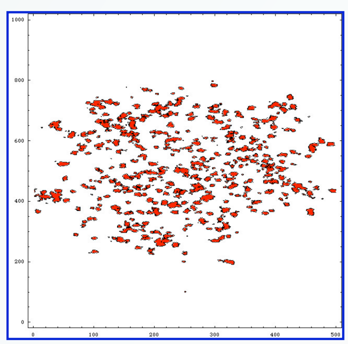
- Delocalized states travel along with current.
- Prediction of superdiffusive spreading of average concentration profile in direction perpendicular to the current for the linear (b=0) case confirmed by simulation.

"Hopping" Bacteria

- At low convection velocities and negative average growth rate, it is expected that the the population should appear to "hop" from one nutrient oasis to another, somewhat like the mechanism of hopping conduction in semiconductors.
- A convection velocity should allow a population to traverse the medium faster in one direction. In particular, if the transit time between oases of separation R is given by some function f(R), then an upper limit on the transit time across a sample is proportional to $f(R_c)$, where R_c is the minimum radius needed to form a system-spanning percolation network. The upper limit with a convection velocity is then given by:

$$T_v \propto f(R_c(1 - v^2\xi^2/D^2)^{\frac{3}{4}})$$

Numerical simulations are being conducted to test this.



Contour plot showing a system with negative average growth rate and no convection after a long time. The bacteria have spread out to occupy any areas of positive nutrient concentration after starting in the middle.

Future Work

- There are some biology groups interested in experimental tests of bacterial transit time in disordered systems.
- A discrete simulation is being written, and the effects of a cutoff are being tested in the continuum code.

References

[1] K.A. Dahmen, D. R. Nelson, and N. Shnerb, *J. Math. Biol.* **41**, 1-23 (2000)
[2] D. R. Nelson and N. Shnerb, *Phys. Rev. E* **58**, 1383-1402 (1998)
[3] B.I. Shklovskii and A.L. Efros, *Electronic Properties of Doped Semiconductors*, (Springer, 1979)
[4] John Carpenter, Ph.D. thesis, available at:
(http://www.physics.uiuc.edu/Research/Publications/theses/copies/Carpenter.pdf)

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