

Perfect Scalability:

From Materials to Informatics and back

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Informatics

From Wikipedia, the free encyclopedia

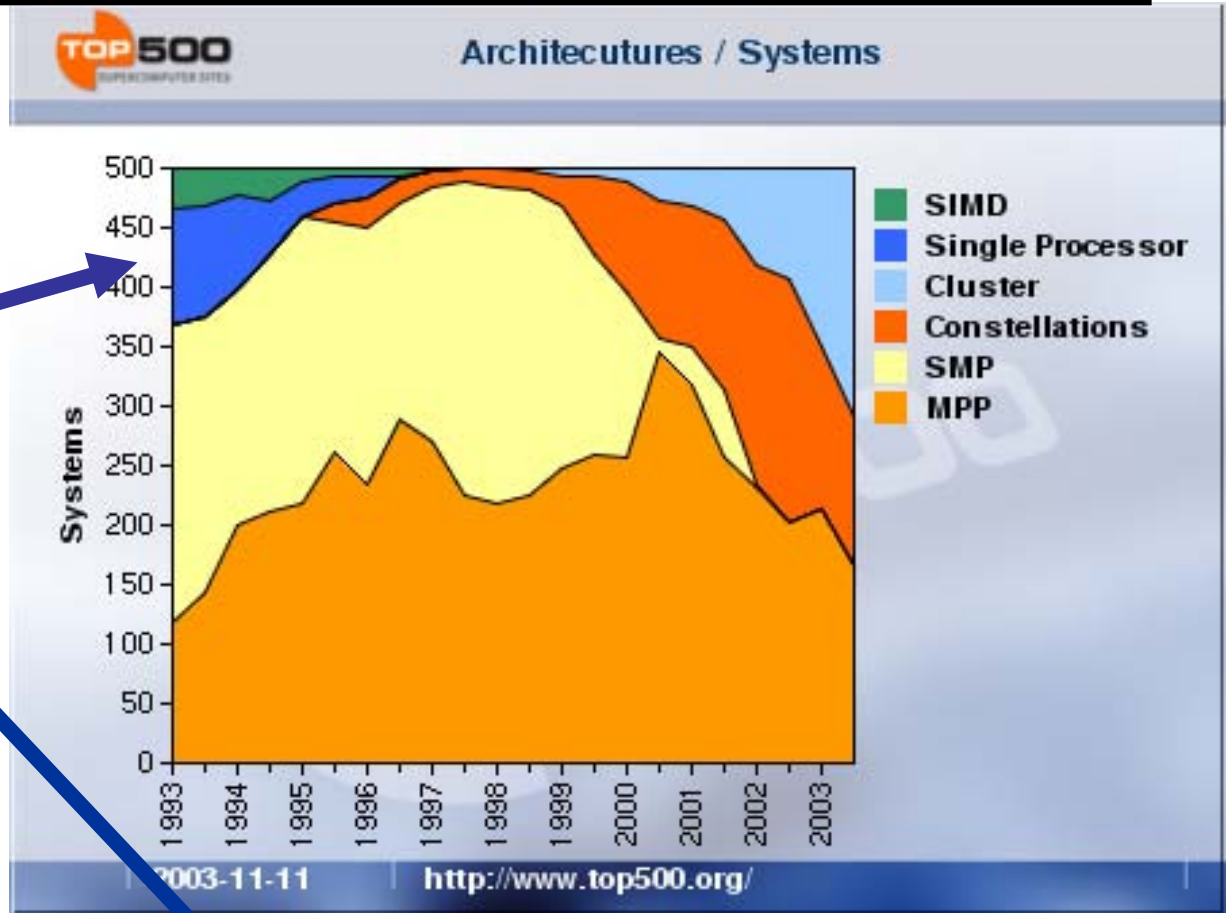
Not to be confused with [Information science](#).

Informatics includes the science of [information](#) and the practice of [information processing](#).

Informatics studies the structure, behavior, and interactions of natural and artificial systems that store, process and communicate information. It also develops its own conceptual and theoretical foundations. Since computers, individuals and organizations all process information, informatics has computational, cognitive and social aspects. Used as a compound, in conjunction with the name of a discipline, as in *medical informatics*, *bio-informatics*, etc., it denotes the specialization of informatics to the management and processing of data, information and knowledge in the named discipline.

Informatics should not be confused with [information theory](#), the mathematical study of the concept of information, or [Library and information science](#) a field related to [libraries](#) and related information fields.

Problem: no signal faster than speed of light

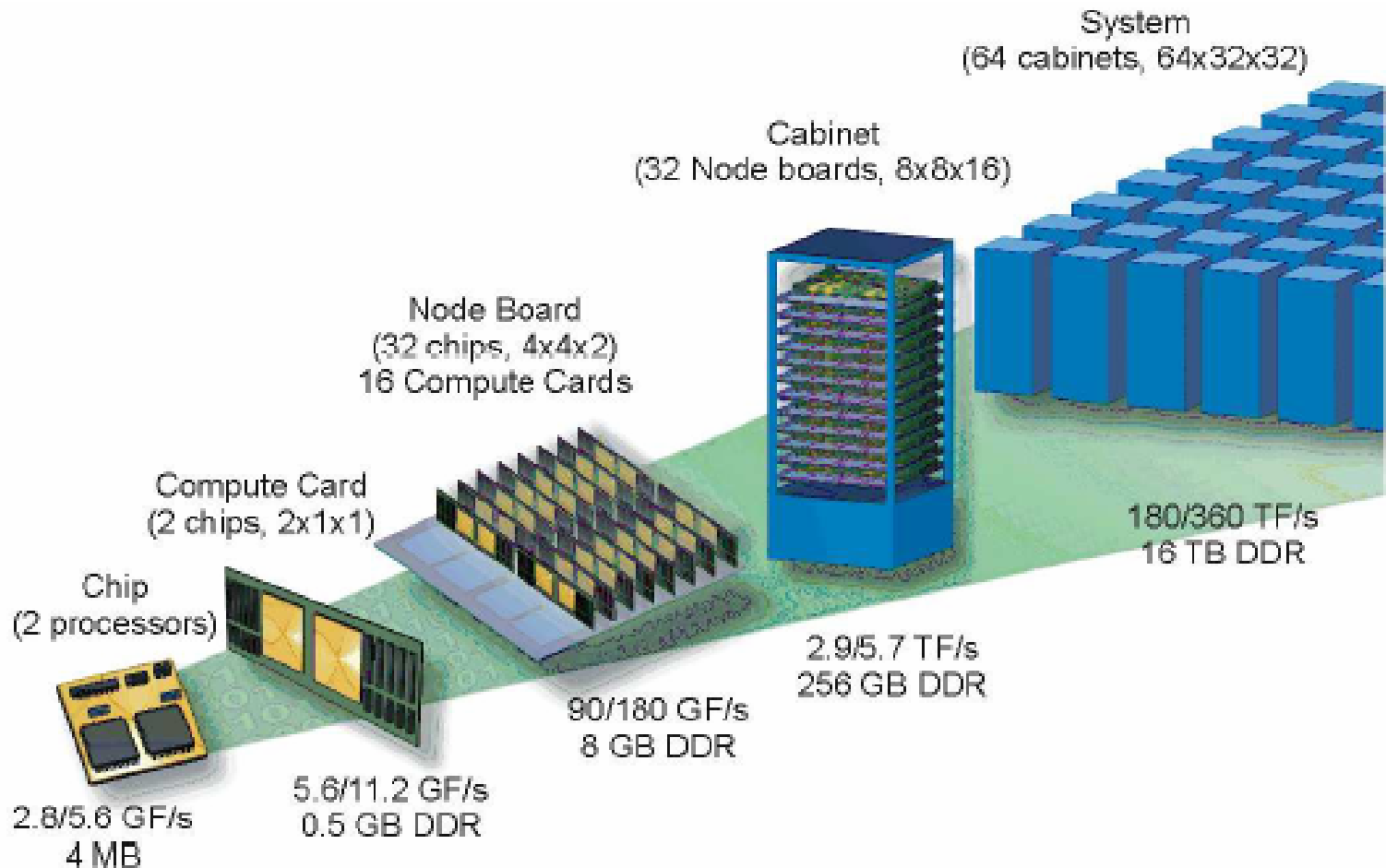


Since 1996 (at least)

computing on supercomputers *requires* parallel computing

Prediction: in 10 years almost all PCs will have >8 PEs
(Processing Elements)

BlueGene/L



$$2 \times 2 \times 32 \times 32 \times 64 = 131,072 \text{ PEs}$$

Perfect Scalability

N_{PE} = # Processing Elements

Non-trivial parallelization

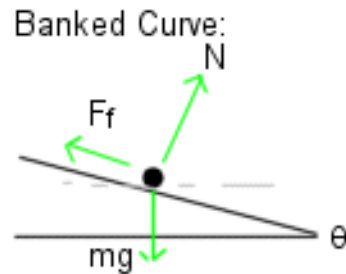
Amount of work $O(N_{PE})$

- For large N_{PE} , utilization independent of N_{PE}
- For large N_{PE} , PE memory independent of N_{PE}
- Number of interconnects $O(1)$ per PE

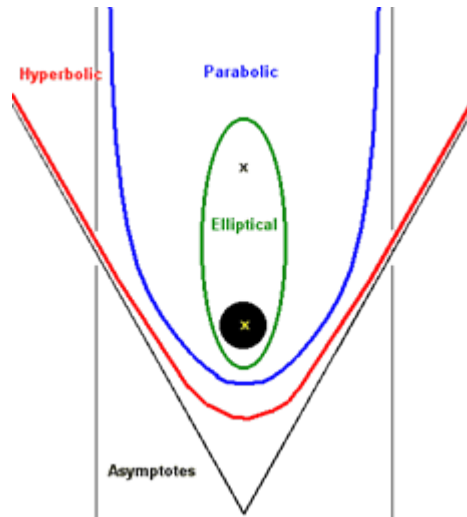


How Computational Physicists Count

1 body



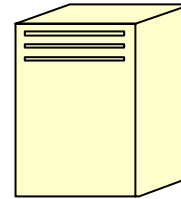
2 body



Too Many bodies

use

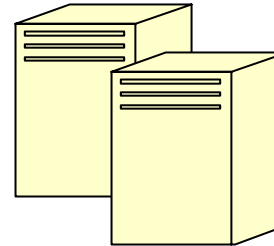
Statistical Mechanics



1 PE (Processor Element)

=

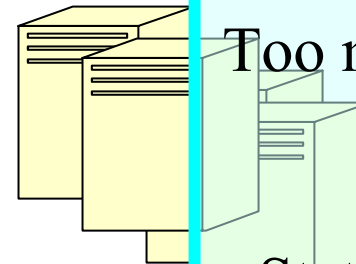
1 grad student



2 PEs

=

2 grad students



Too Many PEs

Too many grad students

use

Statistical Mechanics

Complicated Behavior & Informatics from Nonequilibrium Surface Growth Models



Mississippi State
UNIVERSITY

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Z. Toroczka (LANL → Notre Dame)

P.A. Rikvold (Florida State University)

Z. Rácz (Eötvös University, Budapest)



Motivation for **PDES** model

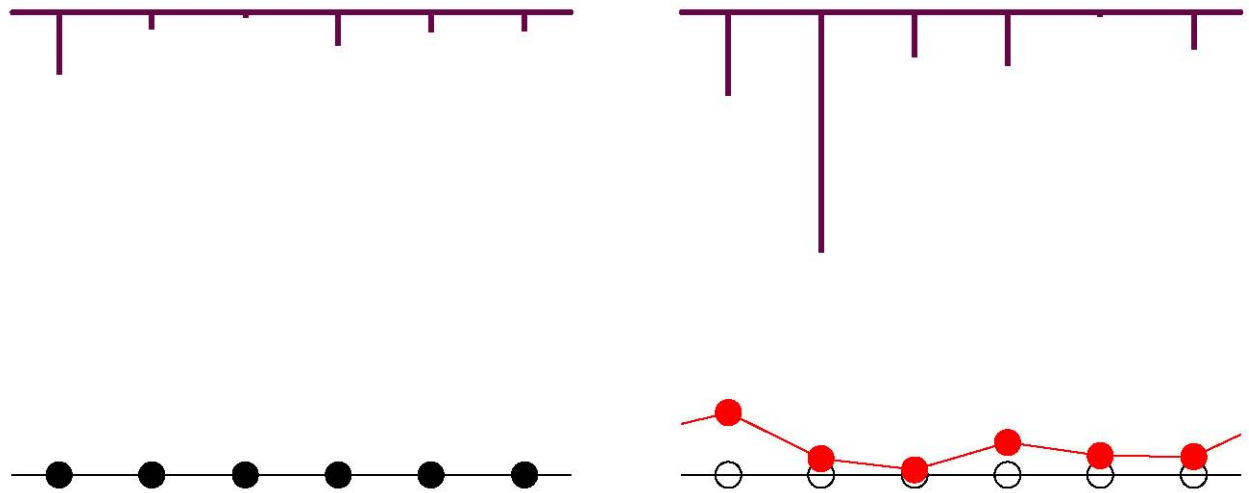
Parallel computing

Non-equilibrium surface growth model: PDES model

$$-\ln(r)$$

$$0 < r < 1$$

Replenish
when
needed



Start with flat interface (*in d dimensions*)

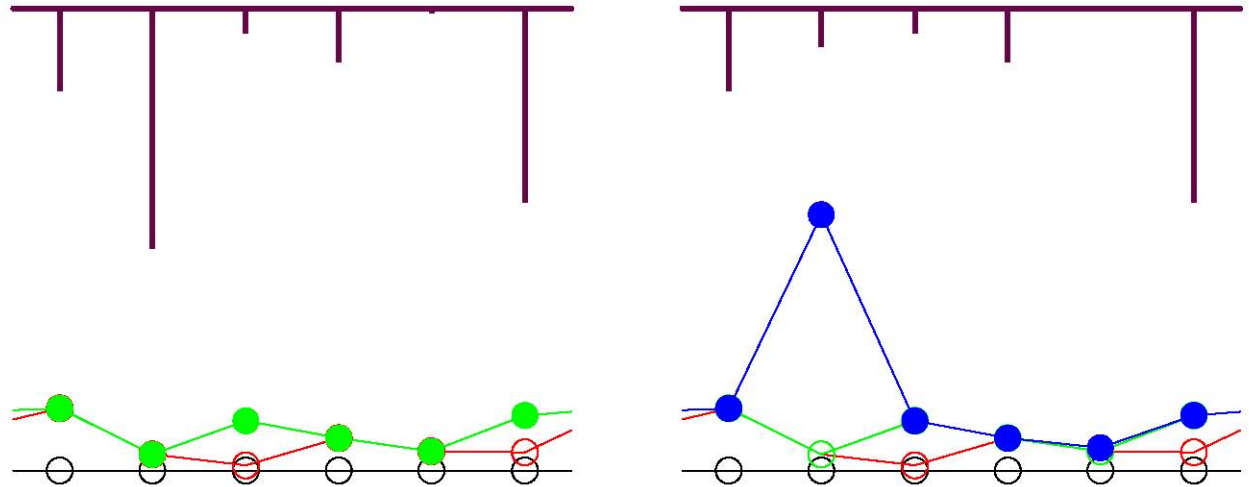
In first step, all ‘drops’ fall

PDES *model*

$$-\ln(r)$$

$$0 < r < 1$$

Replenish
when
needed

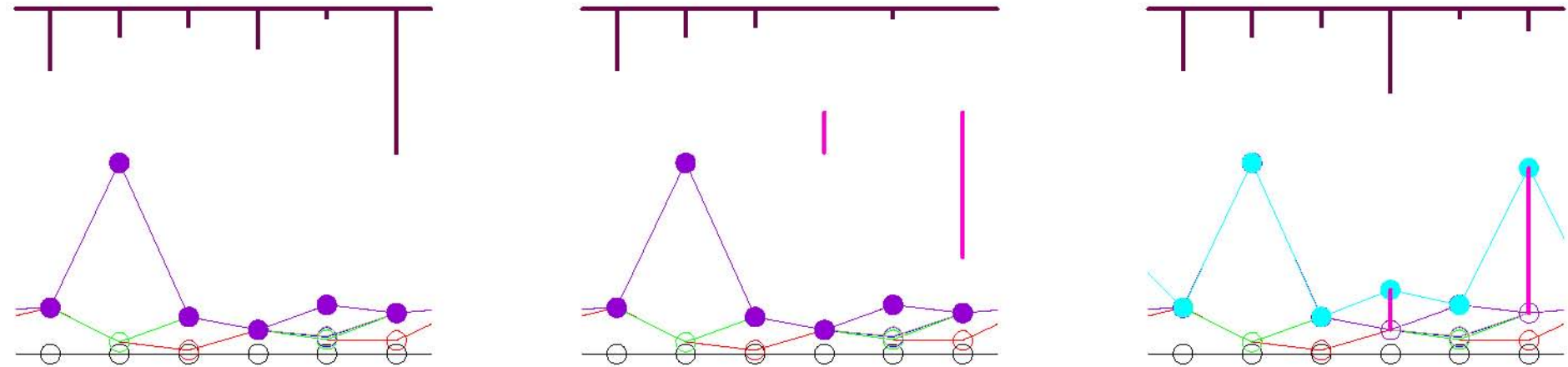


For each step, all ‘drops’ fall ***ONLY*** if the surface underneath is at a local minimum

PDES *model*

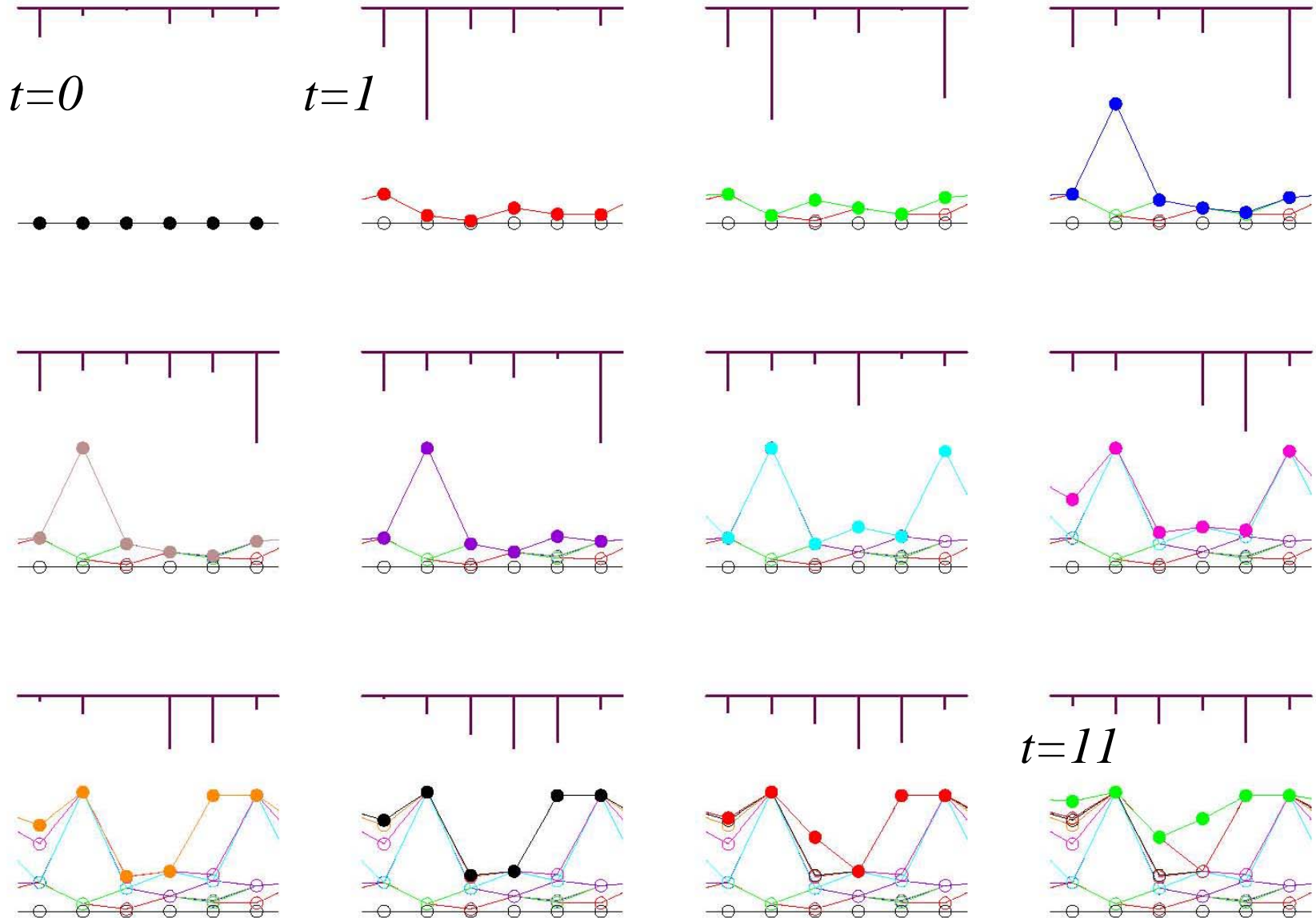
t

$t + 1$



Note: at each step t all 'drops' fall
at the same time

PDES *model*



Discrete Event Simulations



- DES (Discrete Event Simulations)
 - * State changes are discontinuous
 - * Times of state changes are random

PDES

Parallel Discrete Event Simulations

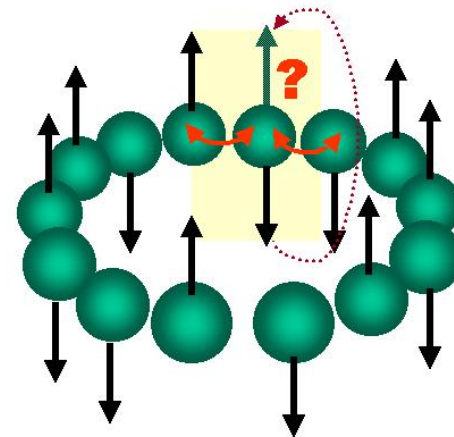
PDES Technology Implications

- **All today's largest computers are massively parallel computers**
- **Must make good use of parallelization in programs for efficiency**
- **Parallel Discrete Event Simulations (PDES)**
 - **Used in military simulations and training ('what-if' scenarios)**
 - **Used in homeland security simulations and training**
 - **Used in modeling of factory deliveries**
 - **Used in modeling temporal drug concentrations in patient models**
 - **Used in simulating materials and materials failure**
 - **Used in modeling switching in cellular and wireless networks**
 - **Used in ecological modeling**
 - **Used in modeling epidemiological models**
 - **Used in electric power grid simulations**

Example:
Dynamic Monte Carlo of Ising spins
with nearest-neighbor interactions

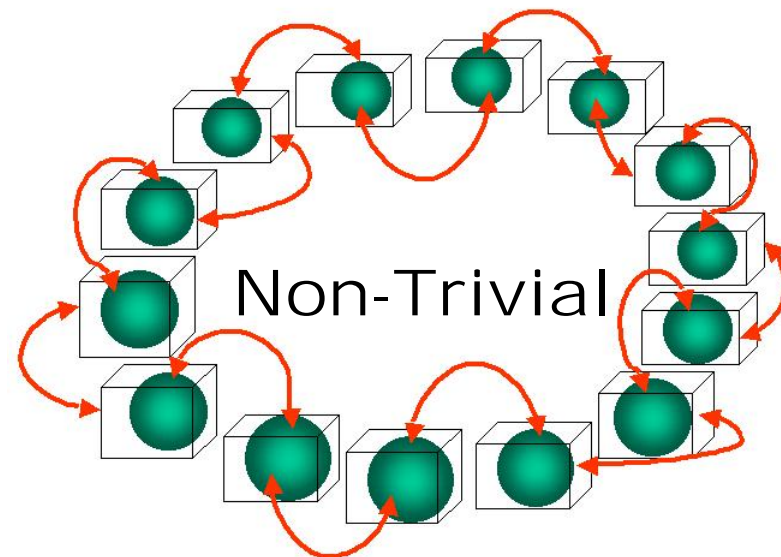
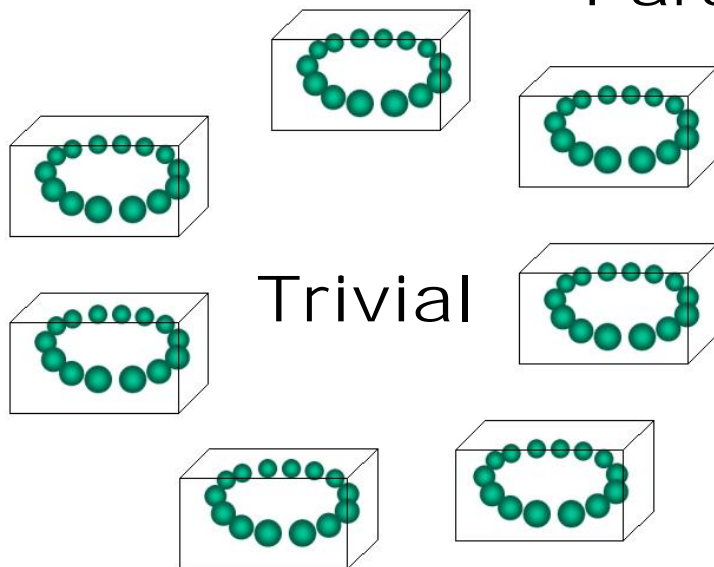
Randomly pick a spin

Decide if spin will be flipped



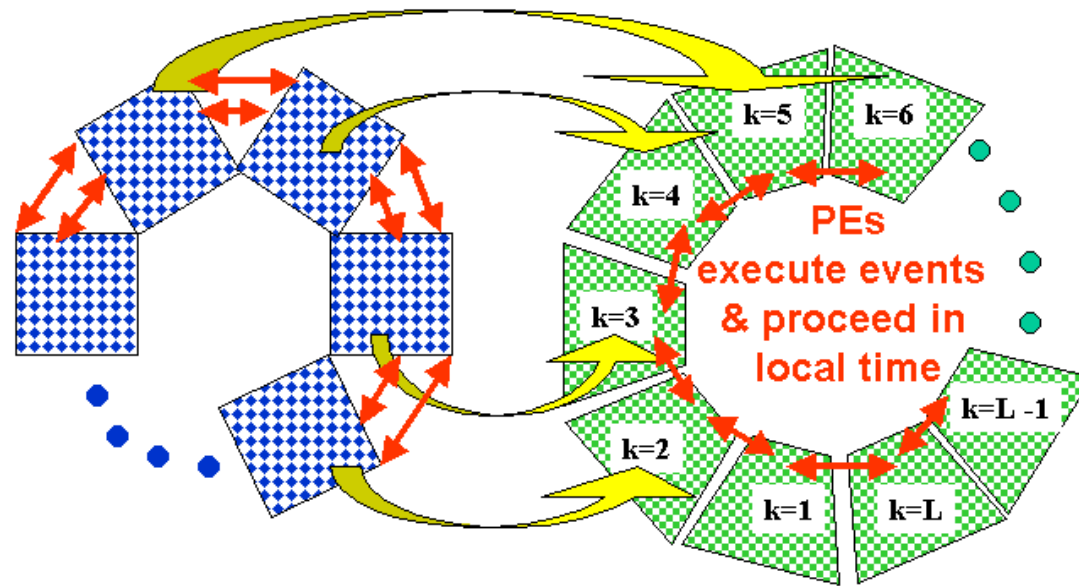
Dynamic Monte Carlo simulations

Parallelization



Physical processes and logical processes

asynchronous
nature of
physical
dynamics



asynchronous
system of
logical
processes

Physical System
spatially extended system of NL
spins, arranged on a lattice

Physical Events/Processes
random spin flipping

Computing System

L PEs: each carries N lattice
sites, N_b of which are border sites

Logical Events/Processes
each PE manages the state of the
assigned subsystem.

discrete event: the spin flip

discrete event: the state update

Parallel discrete-event simulation

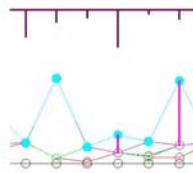
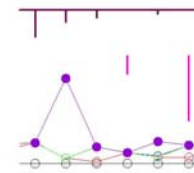
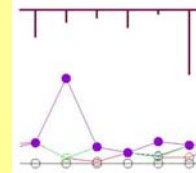
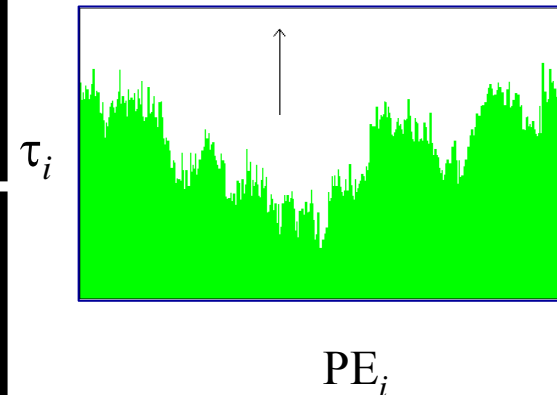
for spatially decomposable **asynchronous cellular automata**

- **Spatial decomposition** on lattice/grid
(for systems with **short-range interactions**
only **local synchronization** between subsystems)
- Changes/updates: independent Poisson arrivals

❖ Each subsystem/block of sites, carried by a processing element (PE) must have its own **local simulated time**, $\{\tau_i\}$ (“virtual time”)

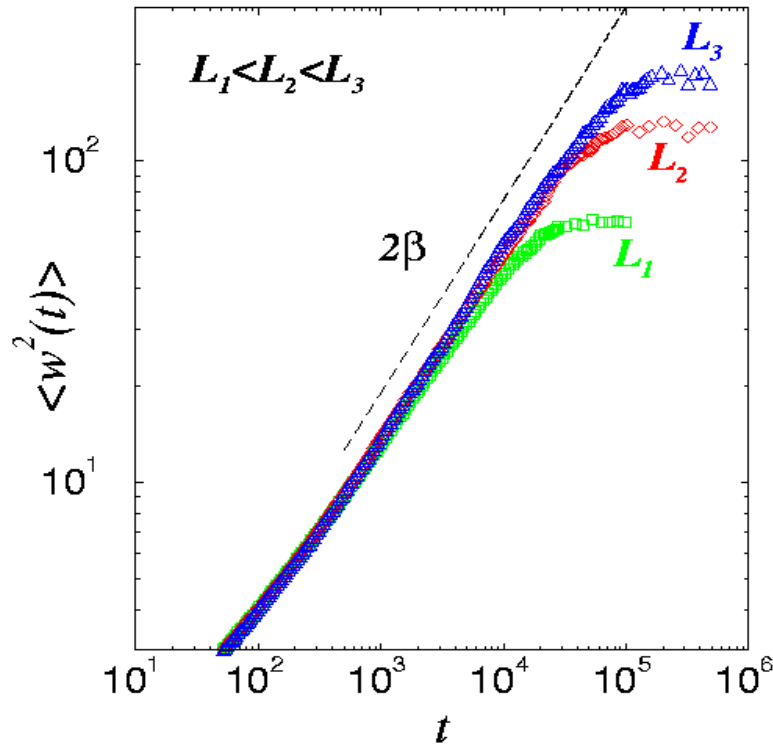
❖ **Synchronization** scheme

❖ PEs must concurrently advance their own Poisson streams, **without violating causality**

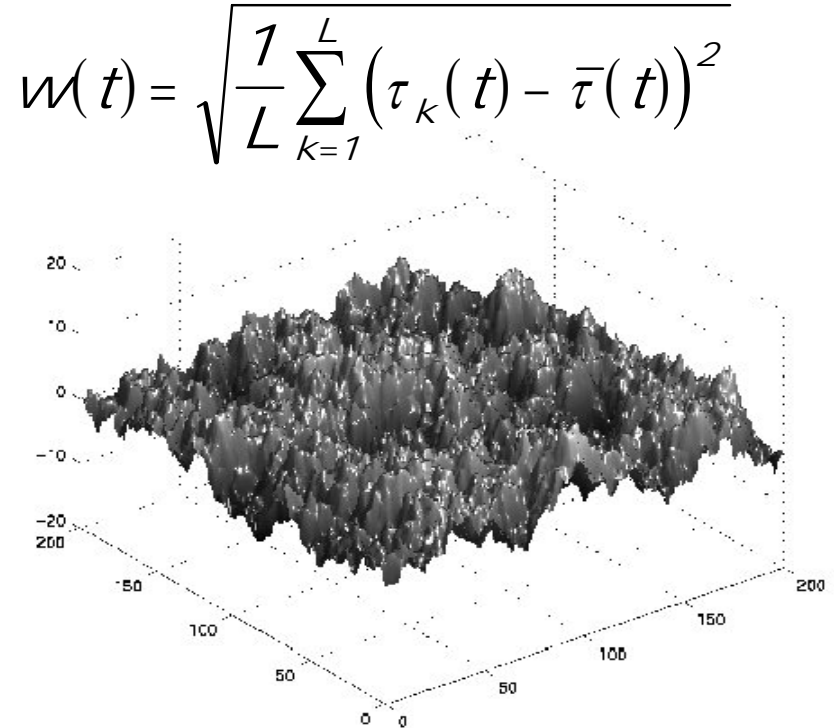


This *is* the **PDES** model

Non-equilibrium surface growth



$$\langle w^2(t) \rangle \sim \begin{cases} t^{2\beta}, & \text{if } t \ll t_\times \\ L^{2\alpha}, & \text{if } t \gg t_\times \end{cases}$$



Dynamic scaling:

$$\alpha = \beta z$$

β growth exponent

z dynamic exponent

α roughness exponent

Coarse graining for the stochastic time surface evolution

Korniss, Toroczkai, Novotny, Rikvold, PRL '00

$$\tau_i(t+1) = \tau_i(t) + \eta_i(t) \Theta[\tau_{i-1}(t) - \tau_i(t)] \Theta[\tau_{i+1}(t) - \tau_i(t)]$$

- $\Theta(\dots)$ is the Heaviside step-function
- $\eta_i(t)$ iid **exponential** random numbers

•
•
•

$$\partial_t \tau = \frac{\partial^2 \tau}{\partial x^2} - \lambda \left(\frac{\partial \tau}{\partial x} \right)^2 + \eta(x, t)$$

Kardar-Parisi-Zhang
equation

$$P[\tau(x)] \propto \exp \left[-\frac{1}{2D} \int dx \left(\frac{\partial \tau}{\partial x} \right)^2 \right]$$

Steady state ($d=1$):
Edwards-Wilkinson
Hamiltonian

❖ Random-walk profile: **short-range correlated local slopes**

“Simulating the simulations”

❖ Universality/roughness

($d=1$)

$$\langle w^2(t) \rangle_L \sim \begin{cases} t^{2\beta}, & \text{if } t \ll t_x \\ L^{2\alpha}, & \text{if } t \gg t_x \end{cases}, \quad t_x \sim L^z, \quad z = \alpha / \beta$$

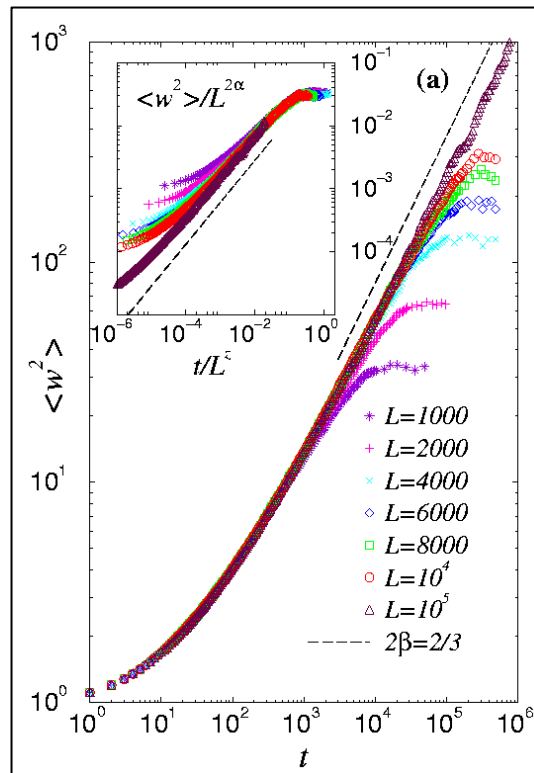
Foltin et.al., '94

$$\beta \approx 0.33, \quad \alpha \approx 0.5$$

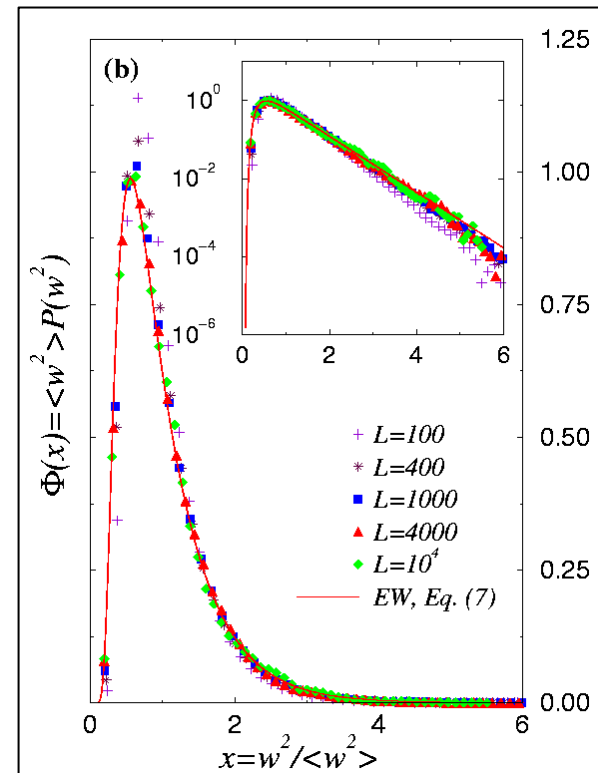
exact KPZ:

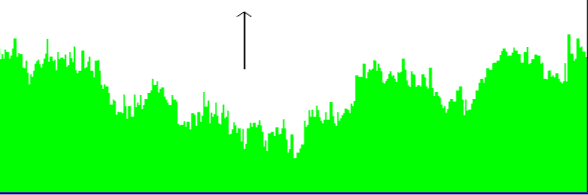
$$\beta = 1/3$$

$$\alpha = 1/2$$



$$P(w^2) = \langle w^2 \rangle^{-1} \Phi(w^2 / \langle w^2 \rangle)$$



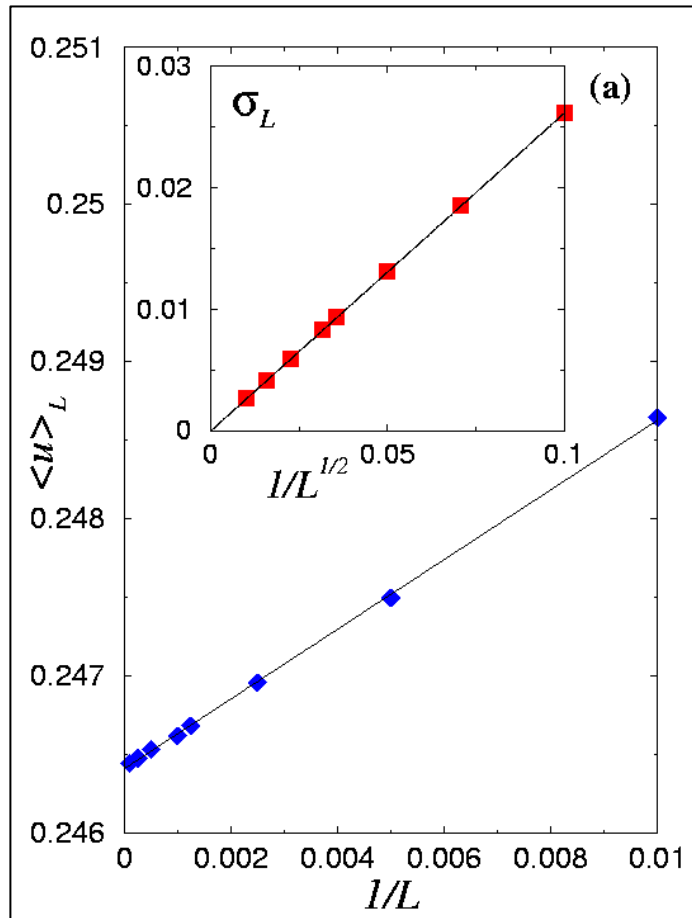


❖ Utilization/efficiency

Finite-size effects for the density of local minima/average growth rate (steady state):

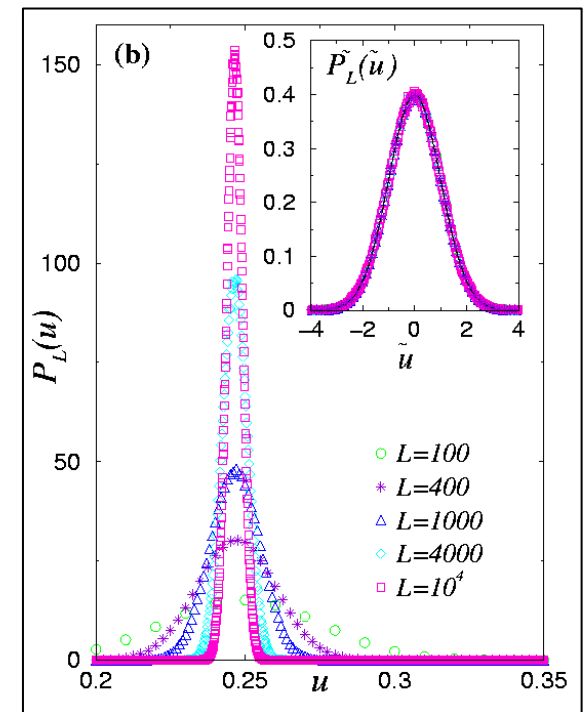
$$\langle u \rangle_L \cong \langle u \rangle_\infty + \frac{\text{const.}}{L}$$

$$\sigma_L = \sqrt{\langle u^2 \rangle_L - \langle u \rangle_L^2} \sim 1/L^{1/2}$$



$d=1$

$$\langle u \rangle_\infty \approx 0.2464$$



Implications for scalability

Virtual Time Horizon belongs to KPZ universality class

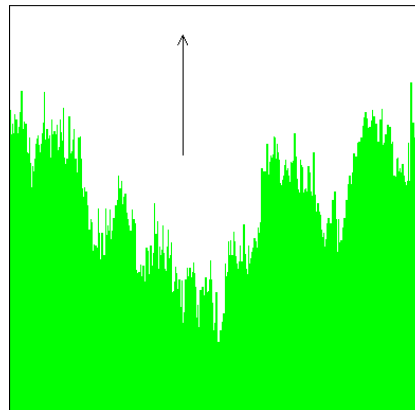
GREAT News ----- **Bad News**

❖ Simulation phase: **scalable**

$$\langle u \rangle_L \cong \langle u \rangle_\infty + \frac{\text{const.}}{L^{2(1-\alpha)}}$$

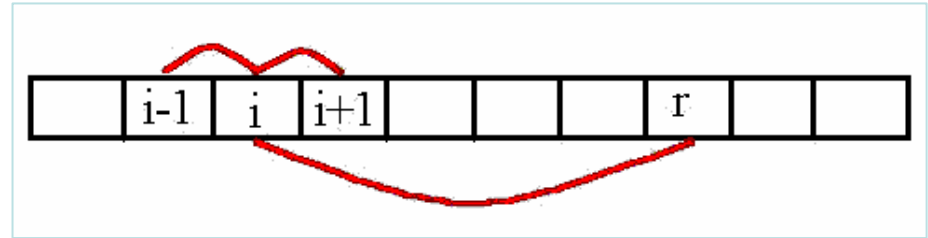
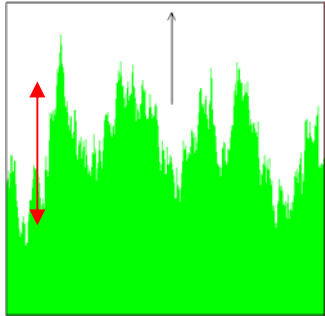
$\langle u \rangle_{\text{clock}}$ asymptotic average rate of progress of the simulation (utilization) is **non-zero**

❖ Measurement (data management) phase: **not scalable**



$$\langle w^2 \rangle_L \sim L^{2\alpha}$$

Quenched random (Small World) connections



United States Patent
Novotny, et al.

6,996,504

February 7, 2006

Fully scalable computer architecture

A scalable computer architecture capable of performing fully scalable simulations includes a plurality of processing elements (PEs) and a plurality of interconnections between the PEs. In this regard, the interconnections can interconnect each processing element to each neighboring processing element located adjacent the respective processing element, and further interconnect at least one processing element to at least one other processing element located remote from the respective at least one processing element. For example, the interconnections can interconnect the plurality of processing elements according to a fractal-type method or a quenched random method. Further, the plurality of interconnections can include at least one interconnection at each length scale of the plurality of processing elements.

Inventors: **Novotny**; Mark A. (Starkville, MS); **Korniss**; Gyorgy (Latham, NY)

Assignee: **Mississippi State University** (Mississippi State, MS)

Appl. No.: **990681**

Filed: **November 14, 2001**

$$w = \text{const.} + \mathcal{O}(L^{-1})$$

$$\tau_i \leq \min \{ \tau_{nn}, \tau_r \}$$

Slopes are still short-range correlated: **non-zero** $\langle u \rangle$

Improve efficiency

Mixing

KPZ + RD

$$\frac{\partial h(x, t)}{\partial t} = \nu \frac{\partial^2 h(x, t)}{\partial x^2} + \lambda \left[\frac{\partial h(x, t)}{\partial x} \right]^2 + D_{kpz} \eta(x, t)$$

+

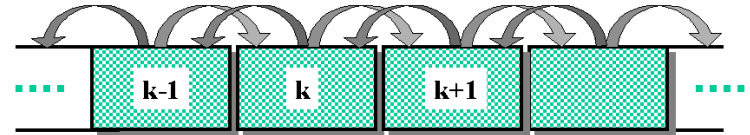
$$\frac{\partial h(x, t)}{\partial t} = D_{rd} \eta(x, t)$$

Simulation model for conservative PDES

Time-step t : index of the simultaneous update attempt

Updates at t : independent Poisson-random processes

If update at t : $h_k(t+1) = h_k(t) + \eta_k(t)$



Update rule

$N=1$

$$h_k(t) \leq \min\{h_{k-1}(t), h_{k+1}(t)\}$$

$N=2$

choose a neighbor

$$h_k(t) \leq h_{nn}(t)$$

$N>2$

choose a lattice site

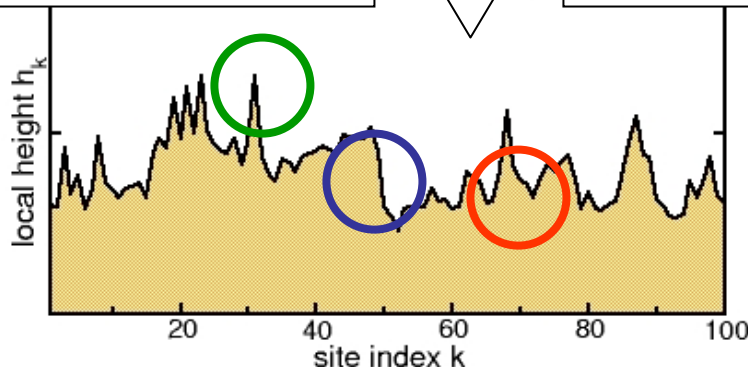
interior

border

$$h_k(t) \leq h_{nn}(t)$$

deposition at t
local height increment δh

update at t
local time increment δh

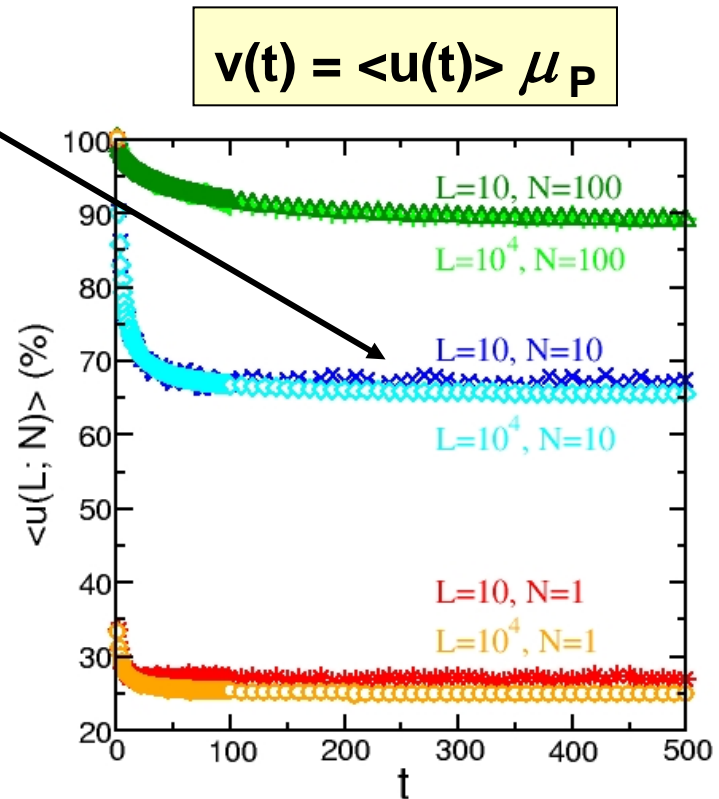
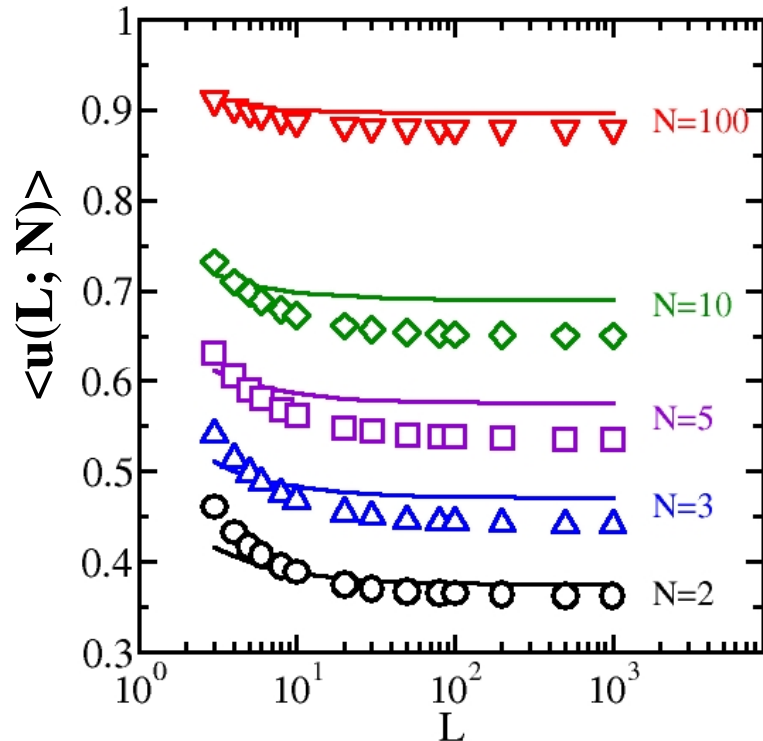


Virtual Time Horizon (VTH)

**Properties of the algorithm
are encoded in the VTH**

Diagnostics: utilization of the parallel processing environment

Steady-state simulations



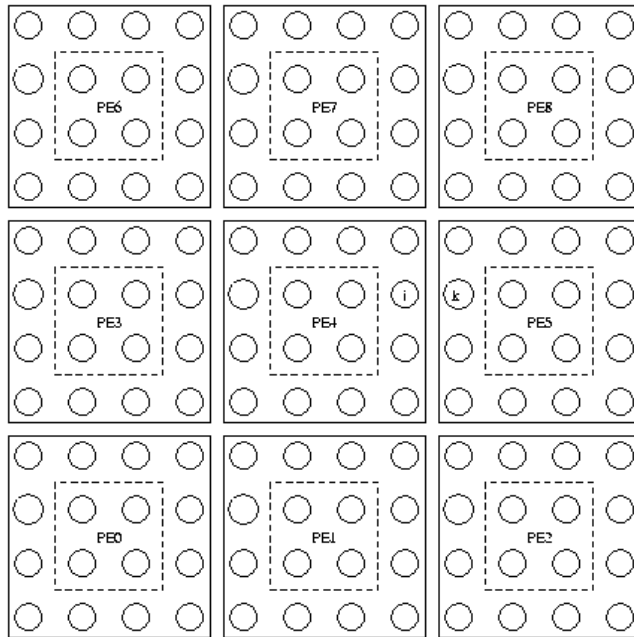
$$\langle u(L > 2; N > 1) \rangle =$$

$$= \left(1 - \frac{1}{\sqrt{2N}}\right) \left(1 - \frac{1}{2\sqrt{2N}} \frac{L-1}{L}\right)$$

PRB **69**, 075407 (2004)

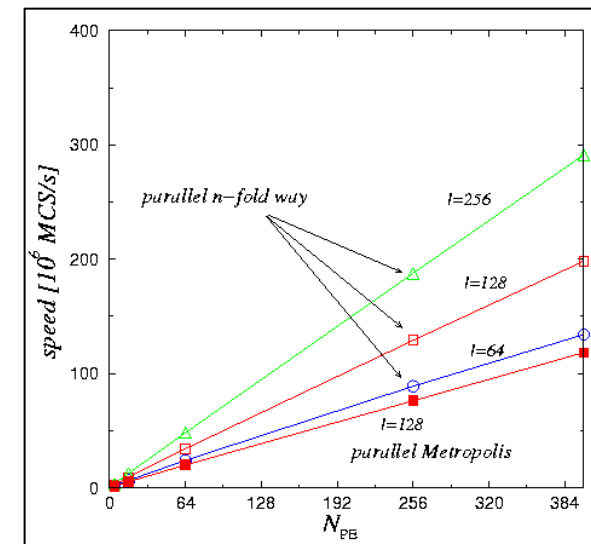
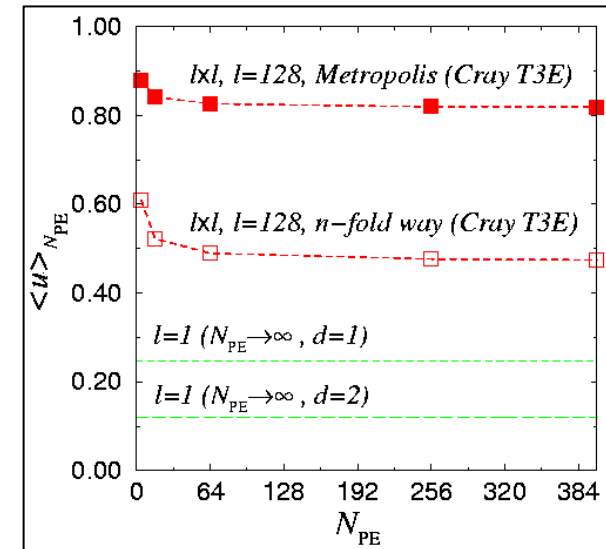
Actual implementation

Dynamics of a thin magnetic film



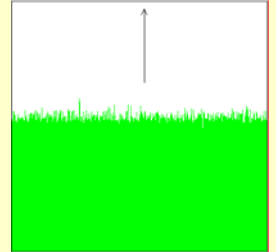
1. Local time incremented
2. Randomly chosen site
3. If chosen site is on the **boundary**,
PE must **wait** until $\tau \leq \min\{\tau_{nn}\}$

$l > 1 \longrightarrow$ Mixing RD+KPZ



PDES Summary and outlook

- Simple **DC** model very useful
- The **tools and methods of non-equilibrium statistical physics** (coarse-graining, finite-size scaling, universality, etc.) can be applied to **scalability modeling and algorithm engineering**
- **Conservative** schemes can be made **perfectly scalable (all short-ranged PDES)**
 - Computational phase always scalable (KPZ universality)
 - Communication phase scalable with small-world network



Discussion

and Provocations

- Neither software nor hardware nor algorithms alone will lead to (non-trivial) perfect scalability
- Without use of statistical mechanics, parallel computing will never be efficient/scalable
- Similar ideas apply to (non-trivial) grid computing
- Similar ideas for sensor networks
- Similar ideas for databases and search
- Similar ideas for fault-tolerant computing
- Similar ideas can be used to design new materials and devices with novel properties



1. **Synchronization in small-world-connected computer networks**, Guclu *et al*, PRE **73**, 066115 (2006).
2. **Universal scaling in mixing correlated growth with randomness**
Kolakowska *et al*, PRE **73**, 011603 (2006).
3. **Desynchronization and speedup in an asynchronous conservative parallel update protocol**, Kolakowska & Novotny, Ch. 6 in “Artificial Intelligence and Computer Science” (Nova Science 2005).
4. **Evolution of Time Horizons in Parallel and Grid Simulations**, L.N. Shchur and M.A. Novotny, PRE **70**, 026703 (2004).
5. **Roughening of the interfaces in (1+1) dimensional two-component surface growth with an admixture of random deposition**, Kolakowska *et al*, PRE **70**, 051602 (2004).
6. **Discrete-event analytic technique for surface growth problems**, Kolakowska and Novotny, PRB **69**, 075407 (2004).
7. **Suppresssing roughness of virtual times in parallel discrete-event simulations**, Korniss, Novotny, Guclu, Toroczkai, Rikvold, SCIENCE **299**, 677 (2003).
8. **Update statistics in conservative PDES**, Kolakowska *et al*, PRE **68**, 046705 (2003).
9. **Algorithmic scalability in globally constrained conservative PDES**
Kolakowska *et al*, PRE **67**, 046703 (2003).
10. **Algorithms for faster and larger dynamic Metropolis simulations**
Novotny *et al*, AIP Conference proceedings, 2003.
11. **Statistical Properties of the simulated time horizon in conservative PDES**
Korniss *et al*, ACM Proceedings, 2002.